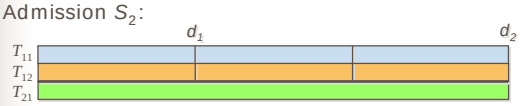
Harmonic QAS – Priority Assignment

- $T = \{T_1,\ldots,T_n\}$ divide into m subsets $S_1,\ldots,S_m$
  $T_i, T_j \in S_k$ iff $d(T_i) = d(T_j)$
- Subsets ordered according to length of period
- Tasks in S_k ordered according to QM
- Priorities in S_k higher than priorities in S_l if $k < l$
- Per S_k: priorities of X higher than priorities of Y

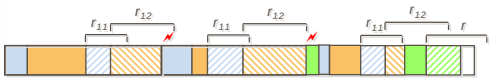
Harmonic QAS – Admission

- Admission S_2:



$$\frac{\sum w_{1i} + \sum r_{1i}}{d_1} + \frac{w_{21}}{d_2} \le 1$$

- Reservation time T_{21}:



$$r_{21} = \min[\, r \mid P(Y_{21} \le r \wedge$$
$$\wedge 3*\min(d_1, \sum X_{1i} + \sum \min(Y_{1i}, r_{1i})) + X_{21} + Y_{21} \le d_2] \ge q_{21}$$

QRMS – Quality-Rate-Monotonic Scheduling

- Task priorities according to RMS
- Reservation time r_i:
  - $r_i' = \min(r \in \mathbb{R} \mid P(X_i + Y_i \le r) \ge q_i)$
  - $r_i = \max(r_i', w_i)$
- Admission test for harmonic periods:

$$\sum_{i=1}^{n} \frac{r_i}{d_i} \le 1$$

QRMS – Non-optimality

X_1, Y_1, X_2, Y_2 identically distributed:

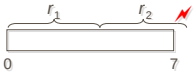
Z	1	2
p	0.5	0.5

$d_1 = d_2 = 7$, $q_1 = q_2 = 0.9$

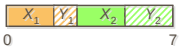
Reservation time:

$X_1 + Y_1$	2	3	4	
p	0.25	0.5	0.25	⇨ $r_1' = r_2' = 4$
Σp	0.25	0.75	1	⇨ $r_1 = r_2 = 4$

Admission test:



But task set is schedulable:



$P(X_1+Y_1+X_2+Y_2 \le 7) = 1 - P(X_1+\ldots+Y_2 = 8) = 1 - 1/16 = 0.9275$

QRMS vs. QAS

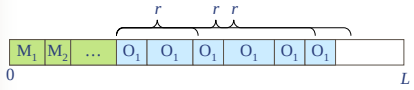
QRMS: + Easy computation of reservation time
+ Easy admission test for harmonic periods
+ Immediately applicable for arbitrary periods
– Not applicable for non-preemptible resources

QAS: + Enables full resource utilization
+ Stronger than QRMS
+ Applicable for more than one optional part
– Not applicable for analytic treatments

– **QRMS & QAS:** Not optimal

QAS – Non-preemptible Resources

- $A_1 = A_1(r)$: number of completed opt. parts of task T_1 within a period



$$P(A_1 \geq k) = P(\sum_{i=1}^{c_1} X_i + (k-1)Y_1 < L \wedge (k-1)Y_1 < r)$$

- Admission (uniform periods):

$$\sum_{i=1}^{n} w_i \leq L$$

$$\exists r_1,...,r_n \in R \; \forall i = 1,...,n : \; r_i = \min(r \in R \mid E(A_i) \geq q_i c_i)$$

Scheduling and Admission Overhead

- Admission time t_{Adm} (QAS, uniform periods)

  complexity $o(v^2)$

  v: number of values of the random variables

v	500	1,000	**2,500**	5,000	10,000	25,000	50,000
t_{Adm}/s	0.015	0.053	**0.304**	1.204	5.017	75.774	609.191
q_{ach}	91.4 %	90.7 %	**90.3 %**	90.3 %	90.1 %	90.0 %	90.1 %

QAS – Static vs. Dynamic Priorities

- Example.

  $X_1 = X_2 = 3$, $Y_1 = Y_2 = 1$; $d_1 = d_2 = 7$; $q_1 = q_2 = 0.1$

- X_i: 2.7 ... 3.3; Y_i: 0.85 ... 1