Real-Time Systems
Marcus Völpp

Hard Real-Time
Multiprocessor Scheduling
Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling (no migration)
- Global Scheduling (task- / job-level migration)
  - G-FP
  - G-EDF
- Optimal MP Scheduling
- MP – Resource Access Protocols
- Open Research Issues
Lessons Learned From UP

- Liu-Layland Criterion for fixed task priority algorithms:
  - can schedule any workload up to a utilization of $U_{RMS}(n) = n^{\frac{n}{\sqrt{2}}} - 1 \leq 0.693$
  - scheduling of workloads with higher utilization not guaranteed

- fixed job priority algorithms are optimal: (e.g., EDF)

- there are optimal greedy algorithms
  - with a single measure characterizing the “importance” of a job (e.g., time to deadline, laxity, …)

- all pre-emptive FTP, FJP algorithms are predictable
  - response times cannot increase when decreasing execution times

- all pre-emptive FTP algorithms and EDF are sustainable
  - no period / deadline anomalies

- simultaneous release is critical instance

- response times depend on set but not on order of high-prio. tasks
Taxonomy of Multiprocessor Scheduling

- Two problems to solve:
  - Priority Problem: When to run a given job of the workload?
    - fixed task priority (e.g., RMS, ...)
    - fixed job priority (e.g., EDF, ...)
    - dynamic job priority (e.g., least laxity, PFAIR, DP-FAIR, ...)

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Taxonomy of Multiprocessor Scheduling

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    - fixed task priority (e.g., RMS, ...)
    - fixed job priority (e.g., EDF, ...)
    - dynamic job priority (e.g., least laxity, PFAIR, DP-FAIR, ...)
  - Allocation Problem: Where to run this job?
    - no migration
    - task-level migration (no migration of running jobs)
    - job-level migration (migrate also running jobs)
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Pre-emption Costs – UP / MP

Migration Costs - MP
Pre-emption Costs

- direct costs:
  - timer / device interrupt
  - save register state
  - manipulate ready list
    - UP: no synchronization required
  - load register state of next thread

- indirect costs
  - cache evictions between two consecutive runs
  - TLB refills after evictions / shootdown
Pre-emption Costs

- direct costs:
  - timer / device interrupt
  - save register state (IA32: 32 – 272 bytes)
  - manipulate ready list
    - UP: no synchronization required
  - load register state of next thread

- indirect costs
  - cache evictions between two consecutive runs
  - TLB refills after evictions / shutdown
Migration Costs

- job-level migration
  - migration of running job implies pre-emption at source CPU

- task-level migration
  - job is already pre-empted

- direct costs
  - manipulate remote / global ready list
    - synchronization
  - fetch register state

- indirect costs
  - fetch active cache working set from remote cache
  - load remaining data from remote memory
Multiprocessor Architectures

- AMD Opteron / Intel Core Duo: SMT + multi core + ccNuma
Multiprocessor Architectures

- Symmetric Multi-Threaded (SMT) Processors
  - $m$ hardware threads
  - shared pipeline
    $m$ HW threads

HW Scheduler (picks one HW Thread at a time)
Multiprocessor Architectures

- Symmetric Multi-Threaded (SMT) Processors
  - \( m \) hardware threads
  - shared pipeline
    - \( m \) HW threads

![Diagram of SMT Processor Architecture]

- HW Scheduler (picks one HW Thread at a time)
- Fetch
- Decode
- Registers
- Execution Unit
- Load / Store Unit
- Write Back / Commit Results

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Multiprocessor Architectures

- Symmetric Multi-Threaded (SMT) Processors
  - operating system multiplexes n SW threads on m HW threads
  - caches + pipeline is shared => no indirect migration costs

![Diagram of a multiprocessor architecture]

- Core
- L1 – D Cache
- L2 - Cache
- Memory
Multi-Core Processors

- operating system multiplexes $n$ SW threads on $m$ cores
- timing of last level cache dominates migration costs
Multiprocessor Architectures

- Symmetric Multiprocessors
  - operating system multiplexes n SW threads on m dies
  - timing of interconnect dominates migration costs

![Diagram of symmetric multiprocessor architecture](image_url)
Multiprocessor Architectures

- (cache coherent) NUMA
  - like SMP
  - non-uniform memory access: fetch from remote memory
Multiprocessor Architectures

- AMD Opteron [Corey: OSDI '08]
Multiprocessor Architectures

- AMD Opteron [Corey: OSDI '08]
Multiprocessor Architectures

- AMD Opteron [Corey: OSDI '08]

![Diagram of AMD Opteron architecture showing memory controllers and levels L1, L2, and L3 with numbers indicating memory allocation and interconnectivity.]
Migration Costs

- Active Cache Working Set
  - cachelines a thread would access again if it would run
  - varies over time
  - ages out after preemption

<table>
<thead>
<tr>
<th>age:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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Migration Costs

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  - ages out after preemption

age: 0 1 2 3
Migration Costs

- **Active Cache Working Set**
  - cachelines a thread would access again if it would run
  - varies over time
  - ages out after preemption

```
age:  0   1   2   3
```

![Diagram showing cache lines and ages]
Migration Costs

- Summary
  - migration costs are highly architecture dependent
  - non-trivial to predict
  - may cause a significant delay when a thread resumes execution

- Assumption for the remainder of this lecture:
  - zero pre-emption and migration costs / attributed to WCET
Design Space of MP Scheduling

- Dynamic job priority / partitioned
- Dynamic job priority / task level migration
- Dynamic job priority / job level migration
- Fixed job priority / partitioned
- Fixed job priority / task level migration
- Fixed job priority / job level migration
- Fixed task priority / partitioned
- Fixed task priority / task level migration
- Fixed task priority / job level migration
Design Space of MP Scheduling

Partitioned
- dyn job prio. / partitioned
- fixed job prio. / partitioned
- fixed task prio. / partitioned

Global
- dyn job prio. / task level migration
- fixed job prio. / task level migration
- fixed task prio. / task level migration
- fixed job prio. / job level migration
- fixed job prio. / job level migration
- fixed task prio. / job level migration
Design Space of MP Scheduling

Relative ordering between classes of scheduling algorithms

- dyn. job prio. / job level migration
- dyn. job prio. / task level migration
- fixed job prio. / partitioned
- fixed job prio. / task level migration
- fixed task prio. / partitioned
- fixed task prio. / task level migration

Later in this lecture
Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling
- Global (Task-Lvl migration) Scheduling
  - G-FTP (e.g., G-RMS)
  - G-EDF
- Optimal MP Scheduling
- MP – Resource Access Protocols
- Open Research Issues
Terminology, Notation and Assumptions

- Periodic Tasks
  - Task \( t_i = (P_i, D_i, C_i) \) \( P_i = \text{const.} \)

- Sporadic Tasks
  - \( P_i = \text{Minimal Interrelease Time} \)

- Deadlines
  - implicit deadline: \( D_i = P_i \) (relative deadline = period)
  - constrained: \( D_i \leq P_i \) (relative deadline < period)
  - arbitrary (deadline may be after period end)

- Utilization
  \[ U_i = \frac{C_i}{P_i} \]

- Density
  \[ \varrho_i = \frac{C_i}{\min(D_i, P_i)} \]
Terminology, Notation and Assumptions

- Assumptions for the remainder of this lecture
  - independent tasks
  - fully pre-emptible / migratable (negligible costs)
  - unlimited number of priorities
  - tasks are single threaded: a job can utilize only 1 CPU at a time
  - jobs do not block (shared resources later in this lecture)
- pictures show schedules for 2 CPUs
Terminology: P-DMS / G-RMS / G-EDF

- Scheduling Algorithms:
  - **Deadline Monotonic Scheduling:**
    - prio inverse proportional to deadline
  - **Rate Monotonic Scheduling:**
    - prio inverse proportional to period
  - **Earliest Deadline First:**
    - job prio. inverse proportional to deadline
### Terminology: P-DMS / G-RMS / G-EDF

- **Scheduling Algorithms:**
  - **Deadline Monotonic Scheduling:**
    - Priorities are inversely proportional to deadlines.
  - **Rate Monotonic Scheduling:**
    - Priorities are inversely proportional to periods.
  - **Earliest Deadline First:**
    - Priorities are inversely proportional to deadlines.

- **P-DMS**
  - Assign threads to processors.
  - Scheduler picks threads from local (per CPU) ready queue.
  - No synchronization overhead for accessing the ready queue.

- **G-RMS / G-EDF**
  - Global:
    - Threads may migrate to other CPUs.
    - Scheduler picks threads from a global ready queue.
    - Accesses to the ready queue must be synchronized.

- **Partitioned**
  - Thread assignment to processors.
  - Scheduler picks threads from local (per CPU) ready queue.
  - No synchronization overhead for accessing the ready queue.
Anomalies

- Simultaneous Release is not Critical Instance [Lauzac '98]
  - longer response time in second period

- yellow misses its deadline

- simultaneous release of all tasks
Anomalies

- Response time (of green) depends not only on set of higher prioritized tasks but also on their relative priority ordering.
Sustainability [Baruah '06]

- A schedulable workload remains schedulable if we
  - decrease the execution time of a task (predictability)
    - otherwise, WCET won't work as admission criterion
  - increase the minimal interrelease time (period) of a task
    - otherwise, more frequent recurrence is no safe approximation
  - increase the relative deadline of a task
    - otherwise, earlier deadline is no safe approximation

- G-FTP + G-EDF are not sustainable if #CPUs > 1

- all pre-emptive FJP / FTP algorithms are predictable

Fixed Job Priority  Fixed Task Priority
Dhall Effect

- The utilization bound of Global EDF is as low as $U_{EDF} = 1 + e$
  - $m$ tasks with short periods and infinitesimal low $U_i$ (e.g., $U_i = e$)
  - 1 task with larger period and $U_j$ close to 1 (e.g., $U_j > (2 - e) / 2$)

- Dhall Effect does not manifest if $U_i < 41\%$
The utilization bound of Global EDF is as low as $U_{EDF} = 1 + e$:

- $m$ tasks with short periods and infinitesimal low $U_i$ (e.g., $U_i = e$)
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- Dhall Effect does not manifest if $U_j < 41\%$
some Impossibility Results

- [Hong '88]
  - No optimal online MP scheduling algorithm for arbitrary jobs unless all jobs have the same relative deadline.

- [Dertouzos '89]
  - Even if execution times are known precisely

- [Fisher '07]
  - No optimal online algorithm for sporadic tasksets with constrained or arbitrary deadlines.
some Impossibility Results

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Partitioned Scheduling

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  - dyn job prio. / partitioned
  - fixed job prio. / partitioned
  - fixed task prio. / partitioned

- **Global**
  - dyn job prio. / task level migration
  - fixed job prio. / task level migration
  - fixed task prio. / task level migration
  - fixed job prio. / job level migration
  - fixed task prio. / job level migration
Partitioned Scheduling

- Split workload by allocating tasks to CPUs
- Run allocated task with UP scheduling algorithm
  - reap benefit of well known UP results
  - optimal task allocation is NP complete:
    - pack \( n \) tasks with density \( d_i \) on \( m \) CPUs with capacity \( d_{\text{max}} = 1 \)
    - Bin-packing

\[ \text{CPU}_0 \rightarrow \text{\( \text{\#} \) \( \text{\#} \) \( \text{\#} \) \( \text{\#} \) \( \text{\#} \)} \]
\[ \text{CPU}_1 \rightarrow \]
\[ \ldots \]
\[ \text{CPU}_m \rightarrow \text{\( \text{\#} \) \( \text{\#} \) \( \text{\#} \) \( \text{\#} \) \( \text{\#} \) \} \]
Partitioned Scheduling

- Utilization bound for implicit deadline workloads [Anderson '01]

\[ U_{opt} = \frac{m + 1}{2} \]

No partitioning scheduling algorithm can produce a feasible schedule of \( m+1 \) tasks with execution time \( 1+e \) and period of 2 on \( m \) processors.
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Easy if blue and green can migrate to CPU_2
Can we improve on Anderson's Utilization Bound?
  - by allowing a few jobs to migrate

- **PDMS** Partitioned Deadline Monotonic Scheduling
- **HPTS** Split Highest Priority Task
- **DS** Allocate according to Highest Density First
\[\begin{align*}
\tau_1 &= (4,3,1) & u_1 &= 0.25 \quad \delta_1 = 1/3 = 0.33 \\
\tau_2 &= (6,2,2) & u_2 &= 0.33 \quad \delta_2 = 1 \\
\tau_3 &= (4,4,1) & u_3 &= 0.25 \quad \delta_3 = 1/4 = 0.25 \\
\tau_4 &= (6,4,2) & u_4 &= 0.33 \quad \delta_4 = 1/2 = 0.5 \\
\tau_5 &= (6,5,1) & u_5 &= 0.16 \quad \delta_5 = 1/5 = 0.2 \\
\end{align*}\]

\[u_{\text{sum}} = 1.33 \implies u_{\text{sum}} / 2 = 0.66\%\]
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\tau_4 &= (6,4,2) & u_4 &= 0.33 & \delta_4 &= 1/2 = 0.5 \\
\tau_5 &= (6,5,1) & u_5 &= 0.16 & \delta_5 &= 1/5 = 0.2 \\
\end{align*} \]

\[
\text{CPU_0} \quad \begin{array}{c}
\tau_1 \\
\tau_2 \\
\tau_3 \\
\tau_4 \\
\tau_5 \\
\end{array}
\quad \begin{array}{c}
u_1 \\
u_2 \\
u_3 \\
u_4 \\
u_5 \\
\end{array}
\quad \begin{array}{c}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4 \\
\delta_5 \\
\end{array}
\quad \text{CPU_1}
\]

\[
\text{u}_{\text{sum}} = 1.33 \Rightarrow \text{u}_{\text{sum}} / 2 = 0.66\%
\]
\[ \tau_1 = (4,3,1) \quad u_1 = 0.25 \quad \delta_1 = 1/3 = 0.33 \]
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\[ \tau_4 = (6,4,2) \quad u_4 = 0.33 \quad \delta_4 = 1/2 = 0.5 \]
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\tau_5 &= (6,5,1) & u_5 &= 0.16 & \delta_5 &= \frac{1}{5} = 0.2 \\
\tau_6 &= (4,3,1) & u_6 &= 0.25 & \delta_6 &= \frac{1}{3} = 0.33 \\
\tau_7 &= (6,2,2) & u_7 &= 0.33 & \delta_7 &= 1 \\
\tau_8 &= (4,4,1) & u_8 &= 0.25 & \delta_8 &= \frac{1}{4} = 0.25 \\
\tau_9 &= (6,4,2) & u_9 &= 0.33 & \delta_9 &= \frac{1}{2} = 0.5 \\
\tau_{10} &= (6,5,1) & u_{10} &= 0.16 & \delta_{10} &= \frac{1}{5} = 0.2 \\
\tau_{11} &= (4,3,1) & u_{11} &= 0.25 & \delta_{11} &= \frac{1}{3} = 0.33 \\
\tau_{12} &= (6,2,2) & u_{12} &= 0.33 & \delta_{12} &= 1 \\
\tau_{13} &= (4,4,1) & u_{13} &= 0.25 & \delta_{13} &= \frac{1}{4} = 0.25 \\
\tau_{14} &= (6,4,2) & u_{14} &= 0.33 & \delta_{14} &= \frac{1}{2} = 0.5 \\
\tau_{15} &= (6,5,1) & u_{15} &= 0.16 & \delta_{15} &= \frac{1}{5} = 0.2 \\
\end{align*}
\]

\[
\begin{align*}
u_{\text{sum}} &= 1.33 \implies \frac{u_{\text{sum}}}{2} &= 0.66\%
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PDMS-HPTS-DS

\[
\begin{align*}
\tau_1 &= (4,3,1) \\
\tau_2' &= (6,2,1) \\
\tau_2'' &= (5,1,1) \\
\tau_3 &= (4,4,1) \\
\tau_4 &= (6,4,2) \\
\tau_5 &= (6,5,1)
\end{align*}
\]

Splitting HP Task into \( D'' - R' = D'' = C' \)

\[ u_2 = 0.33 \quad \delta_2 = 1 \]
\[ u_3 = 0.25 \quad \delta_3 = 1/4 = 0.25 \]
\[ u_4 = 0.33 \quad \delta_4 = 1/2 = 0.5 \]
\[ u_5 = 0.16 \quad \delta_5 = 1/5 = 0.2 \]
\[
\sum u = 1.33 \quad \Rightarrow \frac{\sum u}{2} = 0.66\%
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\( \tau_1 = (4,3,1) \)
\( \tau_2' = (6,2,1) \)
\( \tau_2'' = (5,1,1) \)
\( \tau_3 = (4,4,1) \)
\( \tau_4 = (6,4,2) \)
\( \tau_5 = (6,5,1) \)

\( u_1 = 0.25 \) \( \delta_1 = 1/3 = 0.33 \)
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\( u_3 = 0.25 \) \( \delta_3 = 1/4 = 0.25 \)
\( u_4 = 0.33 \) \( \delta_4 = 1/2 = 0.5 \)
\( u_5 = 0.16 \) \( \delta_5 = 1/5 = 0.2 \)

\( u_{\text{sum}} = 1.33 \Rightarrow u_{\text{sum}} / 2 = 0.66\% \)
• $U_{PDMS-HPTS-DS} = 69.3\%$ if all tasks have a utilization $U_i < 41\%$
Global Scheduling

Partitioned
- dyn job prio. / partitioned
- fixed job prio. / partitioned
- fixed task prio. / partitioned

Global
- dyn job prio. / task level migration
- fixed job prio. / task level migration
- fixed task prio. / task level migration
- dyn. job prio. / job level migration
- fixed job prio. / job level migration
- fixed task prio. / job level migration
Global Scheduling

- Always pick the ready jobs of the \( m \) most “important” tasks
- A task may migrate
  - When a new job of a task is released it may receive a different CPU; once started, a job is no longer migrated
- No need for allocation / load balancing
  - Load balancing is automatic
Global Scheduling – Utilization Bound

- Utilization bound for global fixed-job priority algorithms
  - on m CPUs, G-FJP algorithms cannot schedule m+1 tasks with $C_i = 1 + e$, $P_i = 2$ ($e \to 0$)

\[ U_{\text{OPT}} = \lim_{e \to 0} \frac{(m+1)(1+e)}{2} = \frac{m+1}{2} \]
Global Scheduling - Job Level Migration

Partitioned
- dyn job prio. / partitioned
- fixed job prio. / partitioned
- fixed task prio. / partitioned

Global
- dyn job prio. / task level migration
- fixed job prio. / task level migration
- fixed task prio. / task level migration
- fixed job prio. / job level migration
- fixed task prio. / job level migration

[Red box highlighting the global job level migration]
PFAIR [Baruah '96]

- Divide timeline into equal length quanta
- At each quanta of length \( t \), allocate tasks to processors such that the accumulated processor time is either \([tu_i]\) or \([tu_i]\)
**PFAIR [Baruah '96]**

- Divide timeline into equal length quanta
- At each quanta of length $t$, allocate tasks to processors such that the accumulated processor time is either $[tu_i]$ or $[tu_i]$
PFAIR [Baruah '96]

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PFAIR [Baruah '96]

- Divide timeline into equal length quanta

- At each quanta of length t, allocate tasks to processors such that the accumulated processor time is either \([tu_i]\) or \([tu_i]\)

- PFAIR is optimal for periodic implicit deadline tasksets: \(U_{\text{OPT}} = m\)

- Very high preemption and migration costs
DP-Fair [Brandt '10]

- DP-Fair
  - Optimal scheduler for periodic implicit deadline tasksets with a minimal number of preemptions / migrations?
  
  - Recall [Hong '88]:
    - No optimal MP scheduling algorithm for arbitrary tasksets if not all tasks have the same relative deadline

=>

- deadline partitioning
  - any task’s deadline becomes a deadline for all tasks

- always run zero laxity jobs
  - laxity = time to deadline – remaining execution time
  - zero laxity => job may miss its deadline if it is not run

- jobs who twine themselves around the **fluid rate curve** are somehow in good shape
DP-Fair [Brandt '10]

- Fluid Rate Curve

  work remaining

  fluid rate curve
  actual work remaining curve

  zero laxity event: no more time to run others
DP-Fair [Brandt '10]

- DP-Fair:
  - a family of optimal, deadline partitioning scheduling algorithms
  - Split timeline into chunks according to job deadlines
  - Allocate work to a chunk proportional to $U_i$
    - local execution time: $C_{i,j} = (t_{j+1} - t_j) U_i$
  - Rule 1: run all jobs with zero local laxity
    - jobs with remaining local execution time = time to end of chunk
  - Rule 2: do not run jobs for more than $C_{i,j}$
  - Rule 3: split up idle time proportional to length of chunk
    - allocate at most $S (t_{j+1} - t_j)$ idle time to chunk $j$
Deadline partitioning

Introduce additional releases / deadlines for all jobs whenever there is such an event for one job in the original schedule => chunks
DP-Fair [Brandt '10]

- Idle time

\[ S = m - \sum_{i=0}^{n} U_i \]

Treat idle time as just another job to schedule.
Allocate work proportional to $U_i$.

Allocate execution (and idle) time of a job proportionally to its utilization

$\Rightarrow$ amount of time that this job must run in a given chunk
DP-Fair [Brandt '10]

- Jobs hit their fluid rate curve at the end of each chunk

some arbitrary ordering

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DP-Wrap – a DP-Fair Scheduler

Work allocated to a chunk:
find a schedule
DP-Wrap – a DP-Fair Scheduler

Work allocated to a chunk:
find a schedule

arrange jobs consecutively
DP-Wrap – a DP-Fair Scheduler

Work allocated to a chunk:
find a schedule

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wrap around to obtain schedule for 2\textsuperscript{nd} CPU, ...
DP-Wrap – a DP-Fair Scheduler

Work allocated to a chunk: find a schedule

arrange jobs consecutively

wrap around to obtain schedule for 2\textsuperscript{nd} CPU, ...
DP-Wrap – a DP-Fair Scheduler
Unnecessary migration of red task at chunk boundaries
=> mirror processor assignment of every second chunk
DP-Wrap (mirrored)

\[ m - 1 \text{ migrations per chunk} \]
\[ n - 1 \text{ context switches per chunk} \]
Design Space of MP Scheduling

dyn job prio. / partitioned

fixed job prio. / partitioned

fixed task prio. / partitioned

dyn job prio. / task level migration

fixed job prio. / task level migration

fixed task prio. / task level migration

dyn. job prio. / job level migration

fixed job prio. / job level migration

fixed task prio. / job level migration
Design Space of MP Scheduling

- dyn. job prio. / job level migration
- dyn. job prio. / task level migration
- fixed job prio. / partitioned
- fixed job prio. / task level migration
- fixed task prio. / partitioned
- fixed task prio. / task level migration
- fixed job prio. / job level migration
- fixed task prio. / job level migration
Design Space of MP Scheduling

U_{opt} = m

U_{opt} = (m+1) / 2

fixed job prio. / partitioned = dyn job prio. / partitioned
dyn. job prio. / job level migration

dyn job prio. / task level migration

fixed task prio. / partitioned

fixed job prio. / partitioned
dyn job prio. / partitioned

fixed task prio. / partitioned
dyn job prio. / partitioned

fixed task prio. / task level migration

fixed job prio. / job level migration

A → B => A can schedule any taskset that B can schedule and more
A ↔ B => dominance is not yet known
Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling
- Global (Task-Lvl migration) Scheduling
  - G-FTP (e.g., G-RMS)
  - G-EDF
- Optimal MP Scheduling
- MP – Resource Access Protocols
- Open Research Issues
MP Resource Access Protocols

- UP:
  - Basic Priority Ceiling Protocol BPCP
  - Stack Resource Protocol SRP (Ceiling Priority Protocol CPP)
    - bounded priority inversion: $|CS|$
    - BPCP does not influence unrelated threads

- General Idea:
  - run UP protocol on every CPU of MP system

- Ceiling Priority $i$ of Resource $R_i$: $\hat{R}_i = \max \text{prio}(t_j)$
  - here, priorities have a global meaning

- System Ceiling $\hat{S} = \max \hat{R}_i$ of held resources

- Synchronization Processor: CPU on which $R_i$ is executed
MP Resource Access Protocols

- **UP-BPCP in a nutshell**
  - resource holders inherits priority from blocked threads
  - resource granted if \( \text{prio}(t_j) > \hat{S} \)
  - only the resource holder, which holds a resource with \( \hat{R}_i = \hat{S} \) receives additional resources

- **UP-SRP (actually UP-CPP) in a nutshell**
  - resource holder runs at max \( \hat{R}_i \) of held resources
  - \( \Rightarrow \) only higher prioritized threads may run (acquire resources)
MPCP – first try
MPCP – first try

unrelated threads can affect blocking time
MP Priority Ceiling Protocol (MPCP)

- Run the priority ceiling protocol on each processor
- Treat global resources as jobs with same relative but strictly higher priority than other threads on the sync. CPU
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MP Stack-Resource Protocol (MSRP)

- SRP on every CPU
- Spin non-preemptively while executing a GCS on the sync. CPU
Flexible Multiprocessor Locking Protocol

- works with global scheduling
- access short resources non-preemptively (busy wait if blocked)
- block on Semaphore if long resource is busy
  - Resource holder inherits priority
- resource groups to avoid deadlocks
Performance

- [Gai '03]:
  - MSRP typically outperforms MPCP for short + typically local CSs

- [Brandenburg '08]:
  - comparison of blocking (FMLP) vs. lock-free vs. wait-free
  - non-blocking are preferable for small + simple CSs
  - wait-free / spin-based are preferable for long CSs
  - spin-based almost always outperforms suspension-based algorithms SRP on every CPU
[Brandenburg '11]:

- clustered scheduling: global within the cluster; partitioned in between

Idea:
- Every task helps out resource holders for a bounded time
- Only the n-highest prioritized threads may acquire resources
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

\[ J_1 \text{ preempts } J_3 \text{ who holds resource} \]
\[ => J_1 \text{ becomes priority donor of } J_3 \]
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

J_1

J_2

J_3

R1

J_4

J_5

J_6

J_6 suspends itself because R1 is held by J_3
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

\[ \text{J}_1, \text{J}_2, \text{J}_3, \text{J}_4, \text{J}_5, \text{J}_6 \]

- \text{J}_4\text{ preempts }\text{J}_6\text{ and becomes its priority donor}
- \text{J}_4\text{ runs because }\text{J}_6\text{ blocks on }\text{J}_3
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

J₄ suspends because priority donors are not allowed to acquire resources
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

J_4 executes R2 because after J_5 completes J_6 is one of the k-highest priority jobs
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

J_3 receives R1 after J_3 releases this resource
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster

$J_3$ no longer holds a resource so $J_1$ stops being its donor
Locking for Clustered Scheduling

- 2 clusters; 2 cores each; 3 jobs / cluster
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Open Issues [Burns '09]

- Limited processor Utilization
  - minimally dynamic algorithms
  - novel partitioning approaches
    - increase the guaranteed processing capability; overheads

- Ineffective Schedulability Tests (in particular, sporadic workloads)
  - large gap between feasibility / infeasibility tests
  - identify finite collection of worst-case job arrival sequences

- Overheads
  - migration costs; run queue manipulations; context switching
  - algorithms that permit intra-cluster migration; task-level migr.

- Task Models
  - [Colette] benefit / overhead oriented task models
  - [Edmons] speedup from extra parallelism
Integrating Job Parallelism [Colette]

Task: \( \tau_i \overset{\text{def}}{=} (C_i, P_i, \Gamma_i) \)

\( \Gamma_i \overset{\text{def}}{=} (\gamma_{i,1}, \gamma_{i,2}, \ldots, \gamma_{i,m}) \)

Job with \( j \) processors completes \( \gamma_{i,j} \times t \) units of execution
Runtime Integration

Threads → Threadpools → Apple’s Grand Central Dispatch
C++11 Futures
...

Real-Time Systems, Multiprocessor Scheduling / Marcus Völp
References

- [Burns '09]

- [Colette]

- [Edmonds]

- [Brandt '10]

- [Hong '88]

- [Anderson '01]

- [Lakshmanan '09]
  K. Lakshmanan, R. Rajkumar, J. Lehoczky, “Partitioned Fixed-Priority Preemptive Scheduling for Multi-Core Processors”, ECRTS, 2009
References

- [Fischer '07]

- [Dertouzos '89]

- [Dhall]

- [Baruah '06]
  S. Baruah, A. Burns, “Sustainable Scheduling Analysis,” RTSS, 2006

- [Corey '08]

- [Carpenter '04]

- [Whitehead]

- [Brandenburg '08]
References

- [Gai ’03]

- [Block]

- [MSRP]

- [MPCP]

- [Lauzac ’98]

- [Baruah ’96]

- [Brandenburg’11]