

Real-Time Systems

Time and Order

(following Tanenbaum/Coulouris for Logical and
Kopetz for Physical Time)

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Overview

- Events, computer generated and environmental
- (Real) Time
- The order of events, temporal and causal
- Logical Clocks, 2 versions
- Physical Clocks and their properties
- Global (real) time in distributed systems

Topics

- Can clocks (logical or physical) be used
 - to derive the order of events
 - to identify events
 - to generate events at certain points in time ?
- Which precision can be achieved
 - to measure time ?
 - to measure durations ?
- How and how often have clocks to be synchronized?

Time in Distributed (Real-Time) Systems

- Actions/events/... in distributed real-time systems
 - Concurrent
 - on different nodes
 - must have a consistent behavior / order.
- Consistent order
 - temporal order
 - causal order
- Global Time Base

Events in Computers

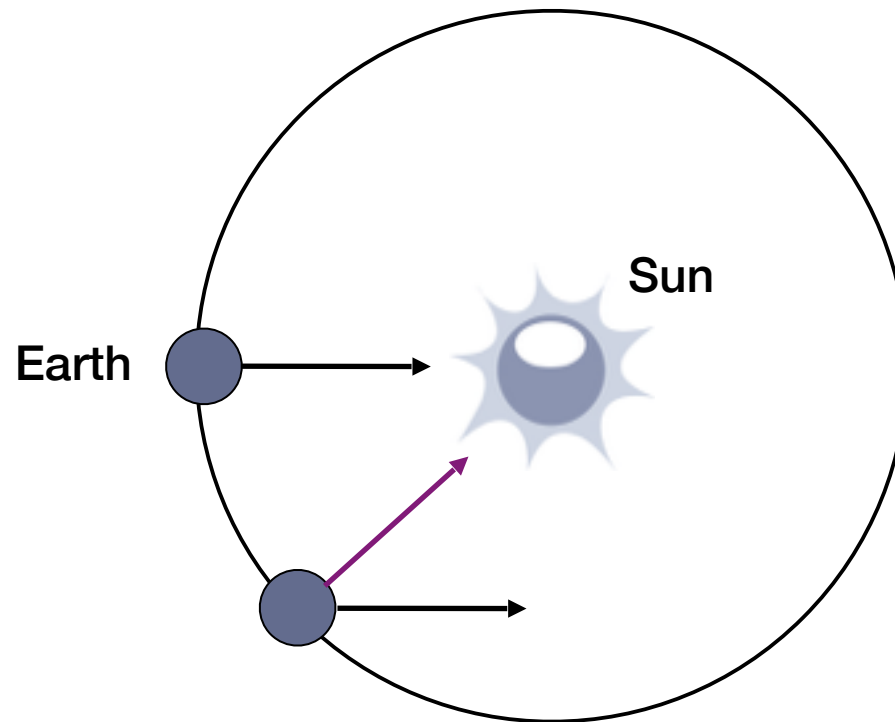
- Computer Generated Events:
 - execution of statement
 - sending/receiving a message
 - start and end of a compilation
 - creation/modification of a file
- Sequence of states is determined by
 - instructions, disk accesses
 - discrete steps

Events in the Real World

- Environmental Events:
 - newton mechanics
 - pipe rupture
 - human interaction
- Sequence of states is determined by
 - laws of physics
 - physical (or real) time: “second”
 - continuous

Astronomical Time

- Solar Day: from noon to noon
- Solar Second: $\text{Solar Day} / (24 * 60 * 60)$



Atomic time

- TAI ... International Atomic Time
- 1 second = “duration of 9192631770 (9 Gigahertz) periods of of the radiation of a specified transition of the caesium atom 133”
- GPS clock is based on TAI

Time Standard(s)

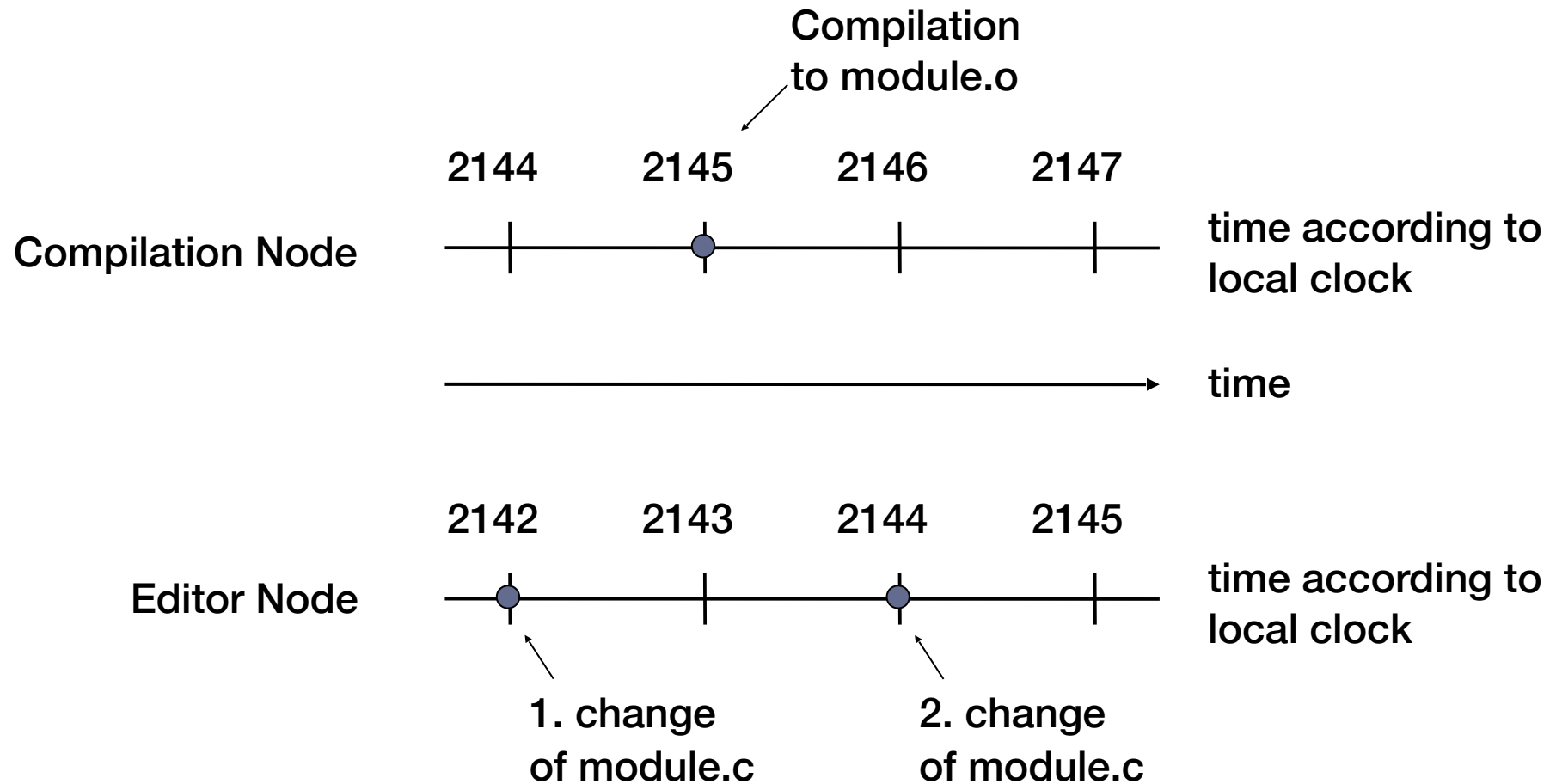
- UTC ... Coordinated Universal Time
 - TAI adjusted with leap seconds to compensate for slowing earth rotation
- Sources:
 - earth-bound radio
 - Geosynchronous satellites
 - GPS

Temporal vs. Causal Order of Events

- Temporal Order:
 - induced by (perfect) timestamp
- Causal Order:
 - induced by some causal dependency between events
- Example
 - e1: somebody enters a room
e2: the telephone rings
 - cases
 - e1: occurs after e2 causal dependency possible
 - e2: occurs after e1 causal dependency unlikely
- Temporal order is necessary but not sufficient to establish causal order.

Another Example

- Imperfect Timestamps can be misleading in establishing causal dependency (example by A.S. Tanenbaum)



Causal Order (for Computer Generated Events)

- Partial Order for Computer Generated Events
 - $a \rightarrow b$ “a causes b”
(happened before, causally dependent)
- 1) If a, b events within a sequential process then $a \rightarrow b$, if a is executed before b.
 - 2) If a is „sending of a message“ by a process and b the „reception of that message“ by another process, then $a \rightarrow b$.
 - 3) $\rightarrow$ is transitive.

Temporal Order

Modeling the continuum of time:

infinite set of instants $\{T\}$

- $\{T\}$ is ordered:
if p, q any 2 instants, then either p, q simultaneous
(i.e. the same instant), or (exclusive) p precedes q ,
or q precedes p
- $\{T\}$ is dense:
at least 1 instant q between p and r
iff p and r are not simultaneous
- Instants are totally ordered

Temporal Order, Timestamps, Duration, Clocks

- Events occur at an instant of the timeline
=> Timestamp.
- Events in a distributed system are partially ordered.
- Duration is a section of the timeline.
- Clocks measure time imperfectly, create imperfect Timestamps.

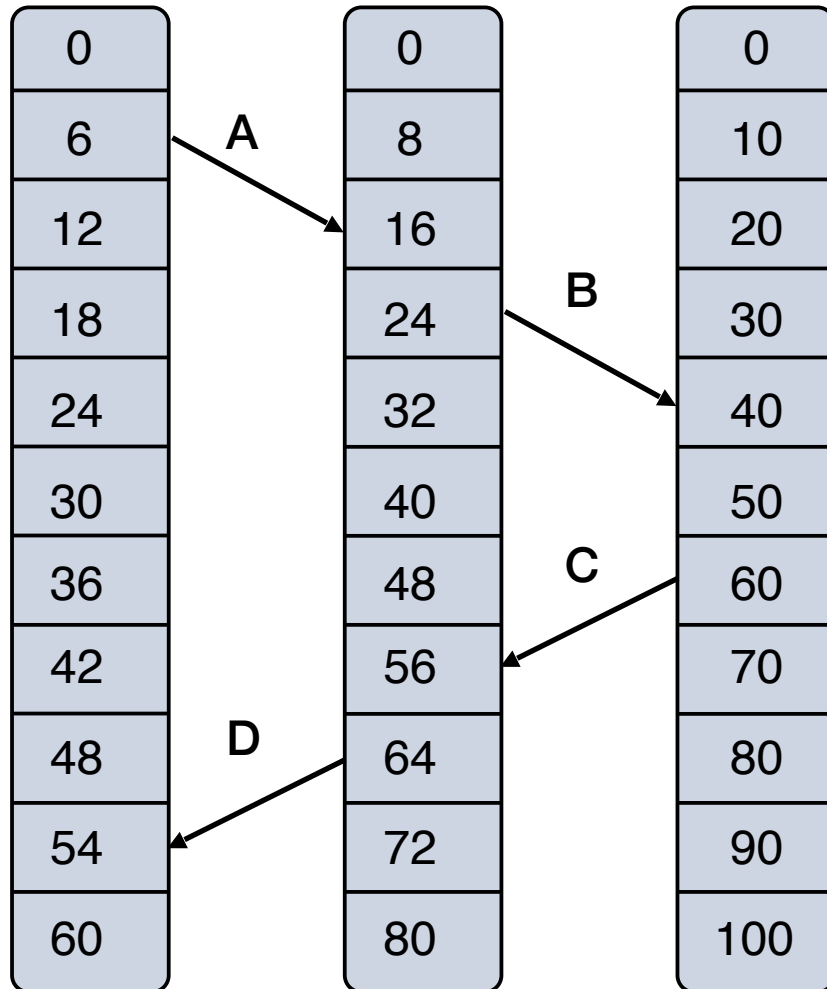
Clocks: Physical and Logical

- Physical Clocks
 - devices to measure time
 - necessarily imperfect (more later)
- Problems:
 - how to create knowledge about causal dependency of computer events without relying on physical clocks? => Logical Clocks
 - how to establish a
 X certainly occurred after **Y** relation (temporal order)
for environmental events? => Global Time

Logical Clocks

- Definitions:
 - monotonically increasing SW counters (COULOURIS)
 - clocks on different computers that are somehow consistent (LAMPORT)
- Events: a, b : $a \rightarrow b$: a causes b (causally dependent)
- Timestamps: $C(a)$, $C(b)$
- Potential Requirements for logical clocks:
 - $a \rightarrow b \Rightarrow C(a) < C(b)$
 - $a \rightarrow b \Leftrightarrow C(a) < C(b)$

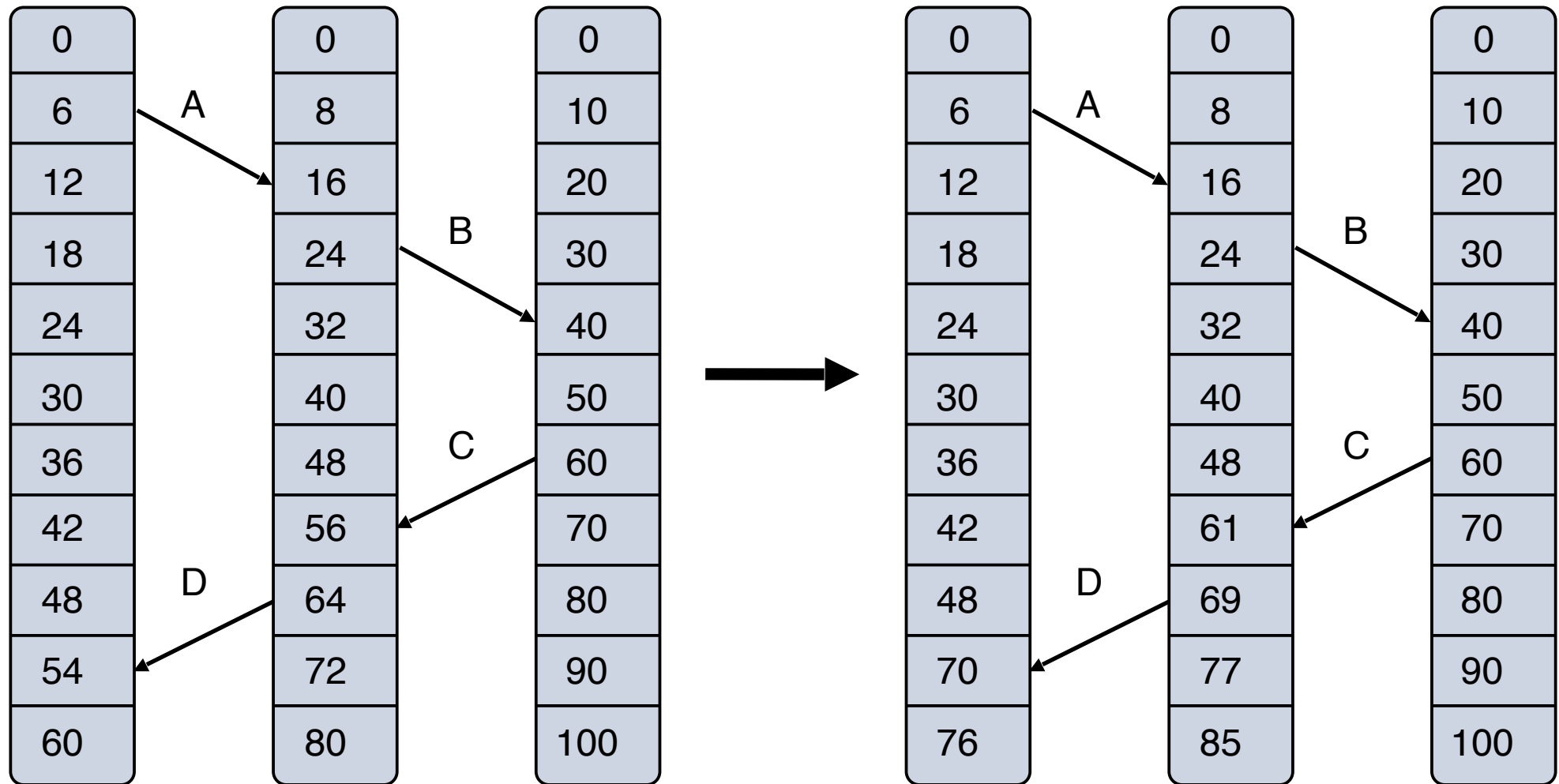
Logical Clock Example



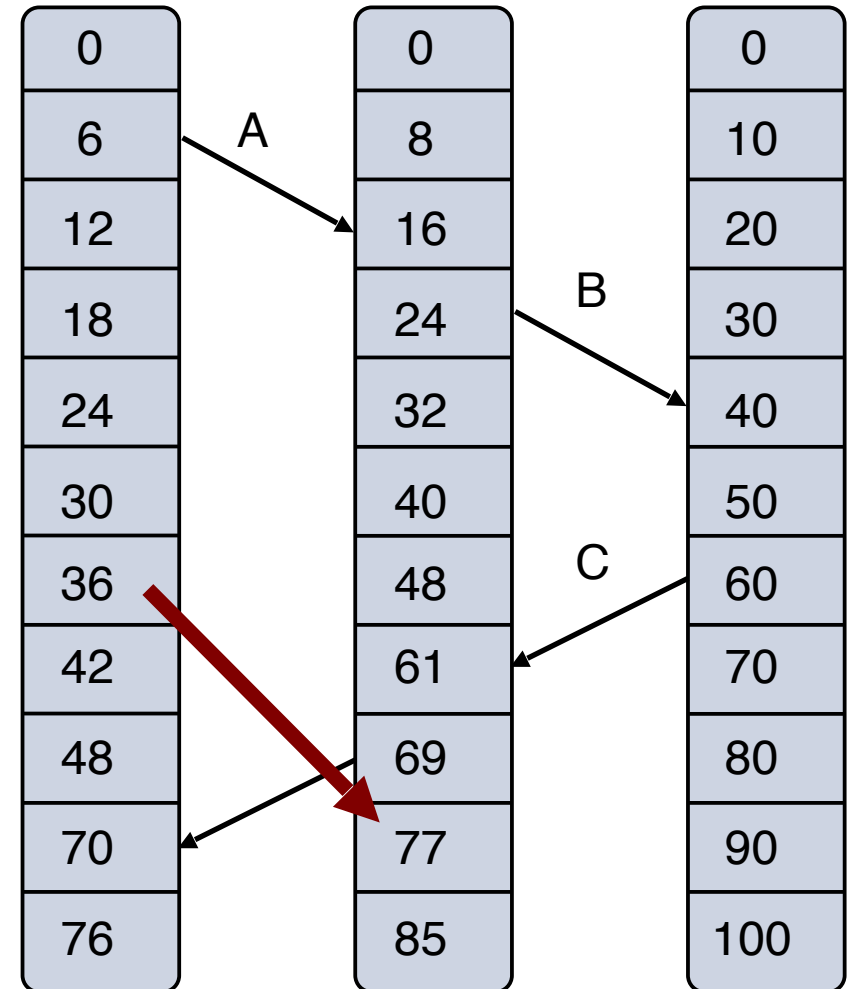
Lamport's Logical Clocks

- each Process has local clock LC_i
- tick:
 - with each local event e :
 $LC_i := LC_i + 1; e$
 - with each sending of a message by process P_i :
 $LC_i := LC_i + 1; \text{send}(m, LC_i)$
 - with each reception of a message (m, LC_m) by P_j :
 $LC_j := \max(LC_m, LC_j); LC_j := LC_j + 1$

Lamport's Logical Clocks



Partial Timestamp Order



Lamport Clocks

- Properties:
- $a \rightarrow b \Rightarrow C(a) < C(b)$,
but not:
 $C(a) < C(b) \Rightarrow a \rightarrow b$
- partial order

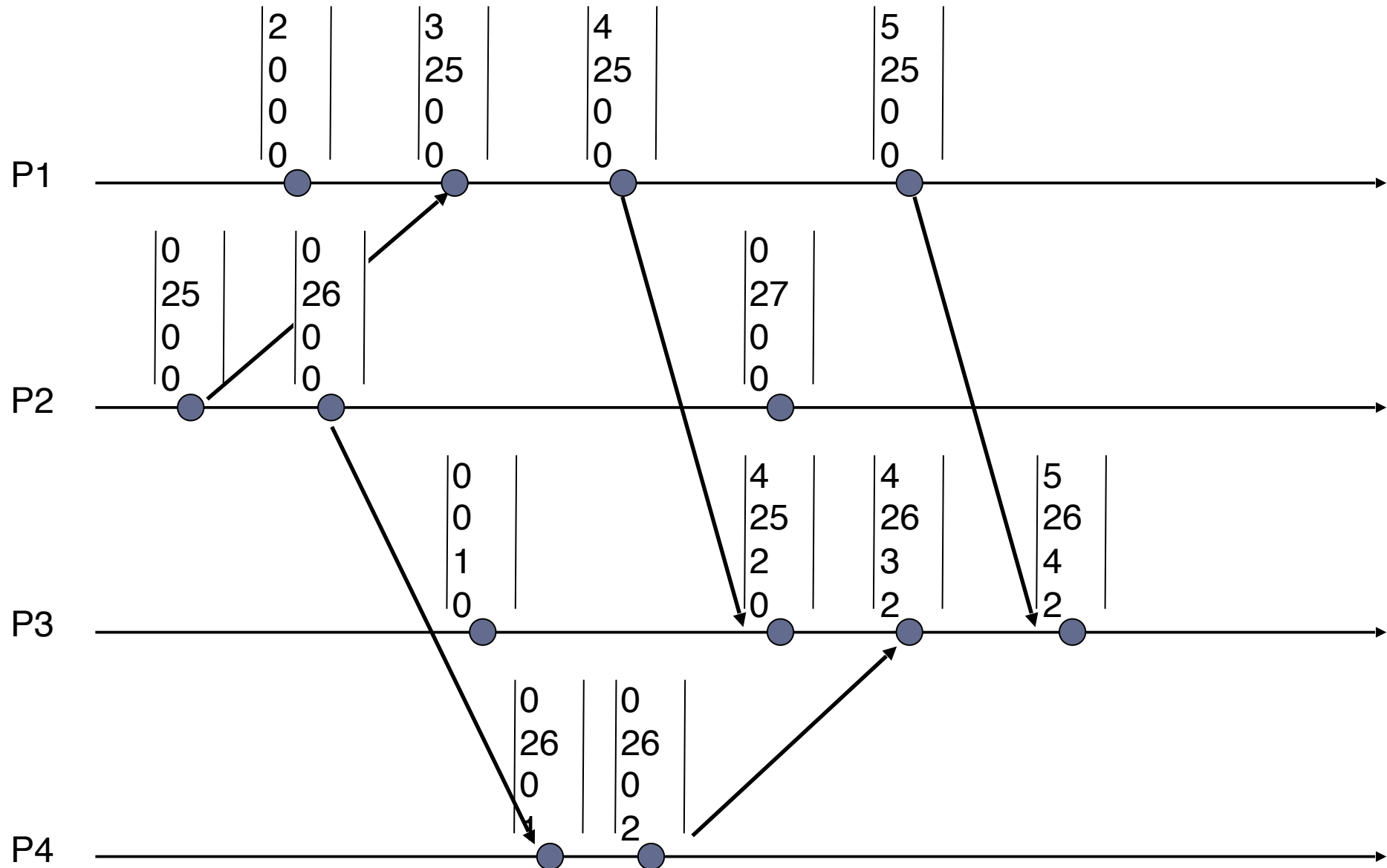
Vector Time (Mattern 1989)

- Each process P_i has its own vector clock C_i .
- C_i : n -dimensional vector (n : number of processes).
- Intuition
- $C_i[j]$: the timestamp of the last event in P_j by which P_i has potentially been effected

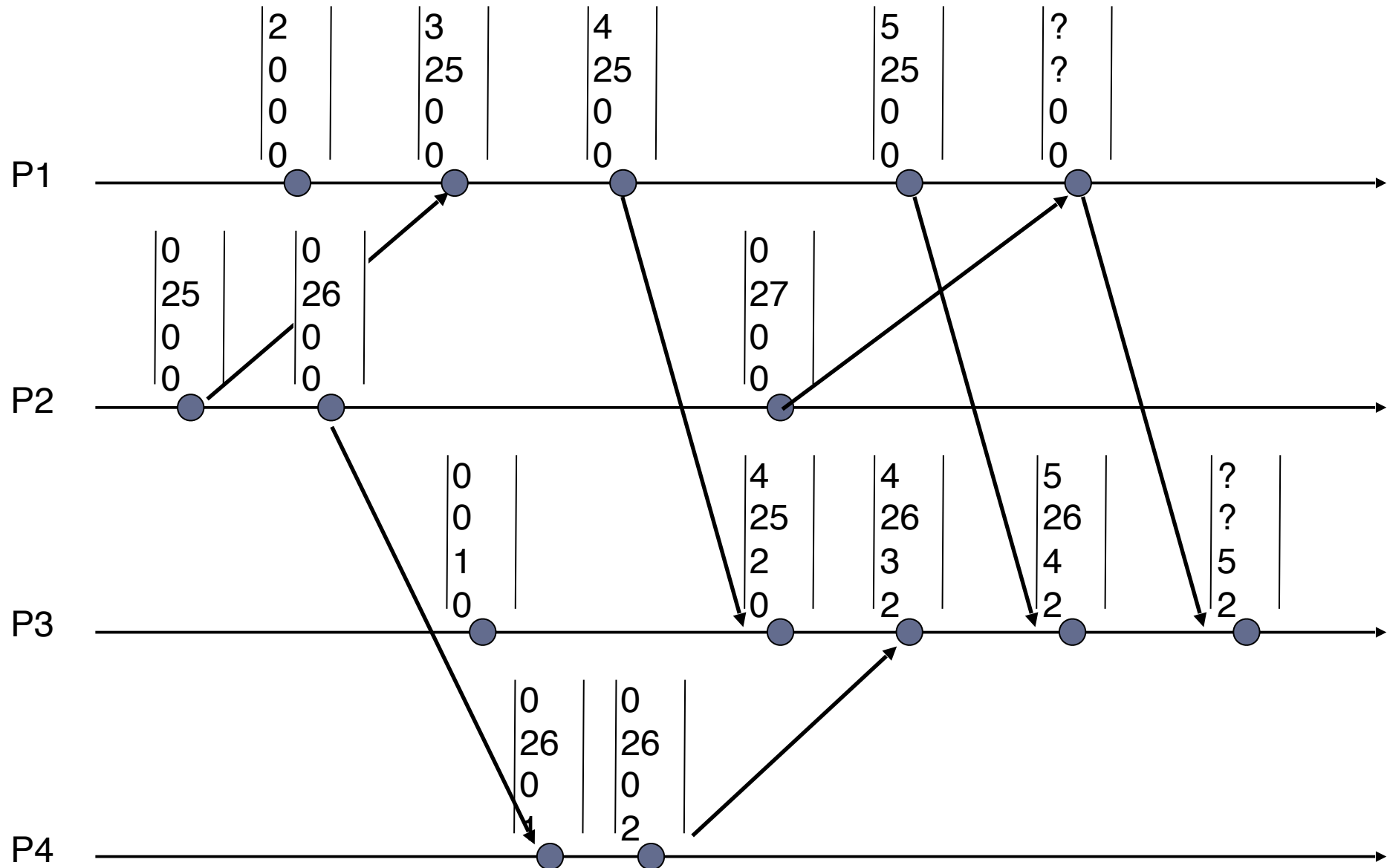
Vector Time Ticks

- Initial:
 $C_i := (0, \dots, 0)$ for all i
- Local event in P_i :
 $C_i[i] := C_i[i] + 1;$
- Sending message m in P_i :
 $C_i[i] := C_i[i] + 1; \text{send}(m, C_i)$
- Receiving a message (m, C_m) in P_j :
 $C_j[j] := C_j[j] + 1;$
 $C_j[k] := \max(C_m[k], C_j[k]), \text{ for all } k$

Example



Example



Properties of Vector Time

- Definition
 - $C_a \leq C_b :\Leftrightarrow \forall k: C_a[k] \leq C_b[k]$
 - $C_a < C_b :\Leftrightarrow C_a \leq C_b \wedge C_a \neq C_b$
 - $C_a \parallel C_b :\Leftrightarrow a \not\prec b \wedge b \not\prec a$
- Property
 - $C_a < C_b \Leftrightarrow a \rightarrow b$

Physical Clocks and Their Properties

Physical Clock

- device for measuring time
- counter + oscillator → “microtick”
- time between microticks:
granularity leads to digitalization error
- Notation:

g^{clock} , $\text{microtick}^{\text{clock}}_{\text{number of tick}}$

- To discuss properties of physical clocks, we invent the perfect reference clock as purely theoretical construct

Reference Clock, Notations (Kopetz)

- Reference Clock z
 - perfect with regard to UTC
 - very small granularity
(to disregard digitalisation error)
 - Reference Ticks: Ticks of the perfect reference clock
- $z(\text{event})$: (Absolute) Timestamp from reference clock
establishes temporal order
- g^k granularity of clock k in microticks of ref. clock

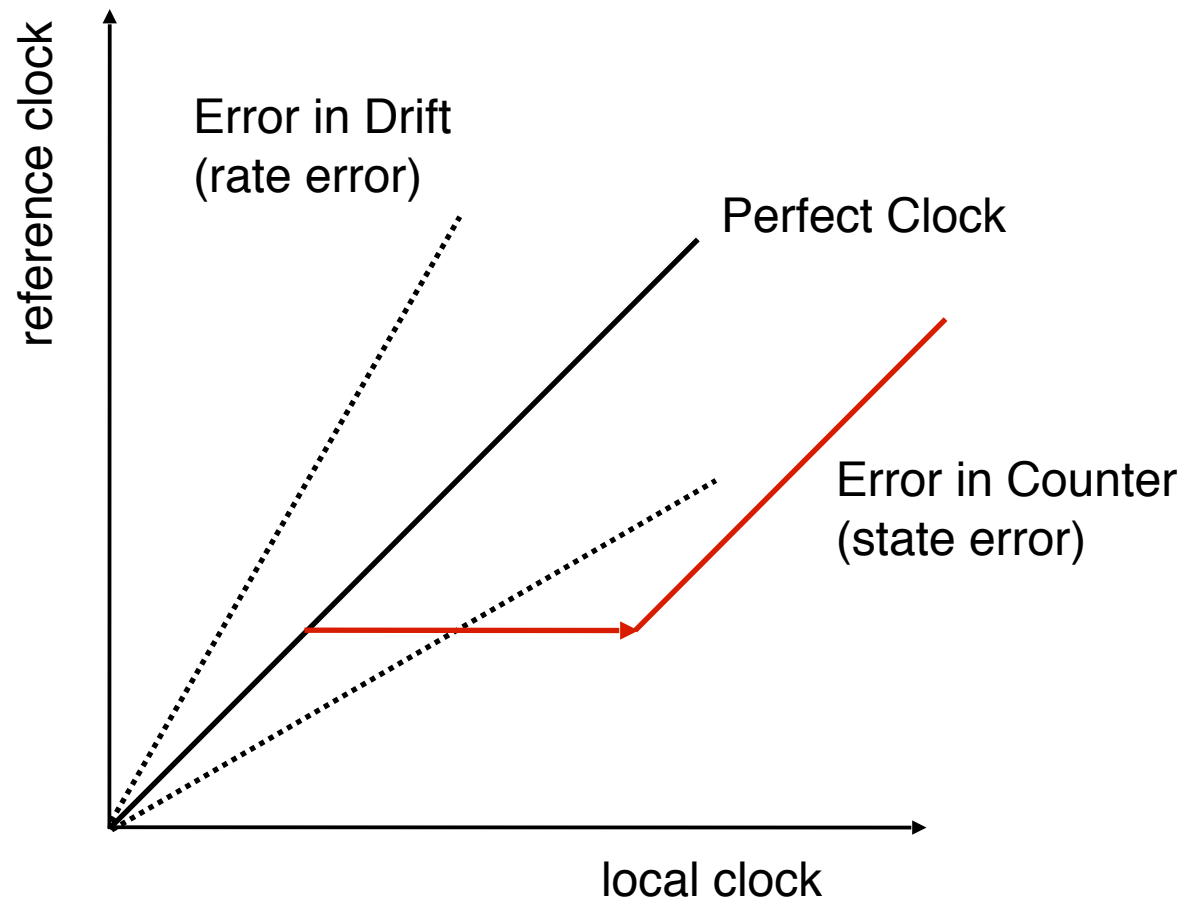
Reference Clock, Notations (Daum)

- Reference Clock z
 - perfect with regard to UTC
 - dense (no ticks, to avoid digitalisation error)
- $z(\text{event})$: (Absolute) Timestamp from reference clock establishes temporal order
- g^k granularity of clock k in terms of z -durations as specified (vs. real behavior)

Tick Tack Terms

- Micro Ticks Ticks generated by the physical oscillator of a clock
- Macro Ticks Multiple of Micro Ticks chosen by designer of clock
- $t^k(\text{event})$ Timestamp in number of microticks of clock k
- Granularity distance between adjacent microticks

Failure Modes: Drift and Counter Errors



Maximum Drift Rate

Drift-Rate

- Varying
- Influenced by environmental conditions (temperature,...)
- clocks specify maximum drift rate ($10^{-2} \dots 10^{-7}$)

$$p_i^k = \left| \frac{z(\text{microtick}_{i+1}^k) - z(\text{microtick}_i^k)}{g^k} - 1 \right|$$

Precision of an Ensemble of Clocks

Offset

- between two clocks j, k of same granularity at microtick i :

$$offset_i^{jk} = |z(microtick_i^j) - z(microtick_i^k)|$$

- in the period of interest: $offset^{jk} = \max_i (offset_i^{jk})$

Precision

- of an ensemble of clocks $\{1, 2, \dots, n\}$ in the period of interest:

$$\Pi = \max_{1 \leq j, k \leq n} (offset^{jk})$$

- maximum offset for any two clocks

Accuracy

Accuracy

- of a given clock in the period of interest:

$$accuracy^k = \max_i |microtick_i^k - i \cdot g^k|$$

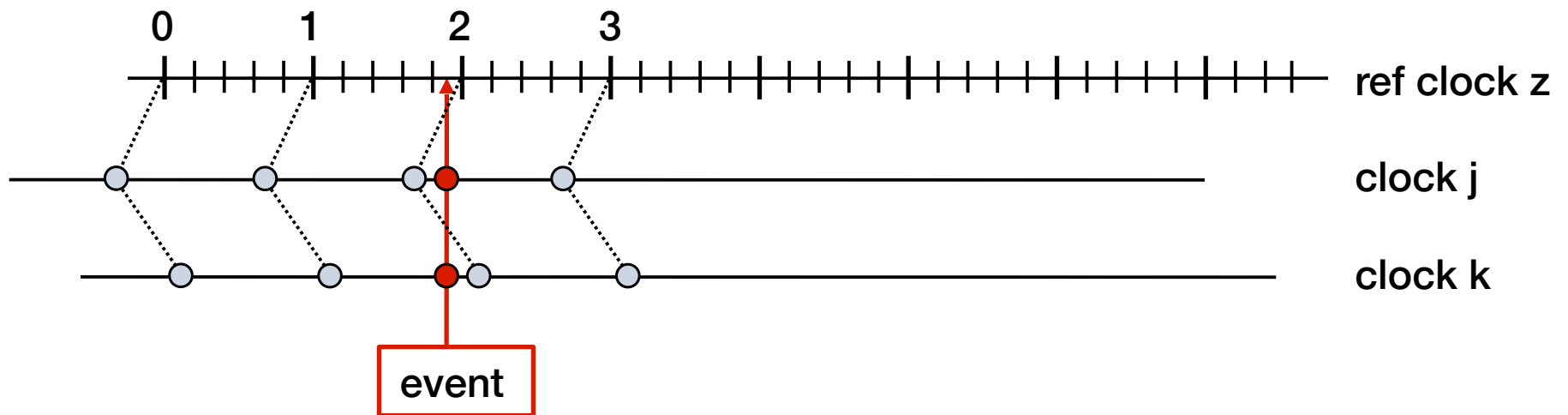
- maximum offset to reference clock
- If all clocks of an ensemble have accuracy A, the precision of the ensemble is ??

Resynchronisation

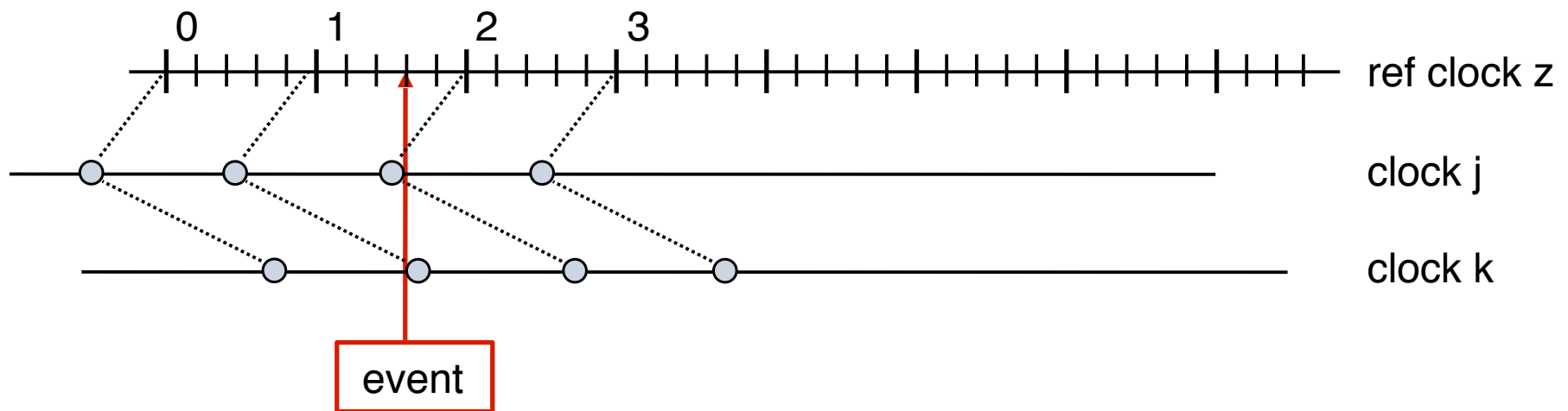
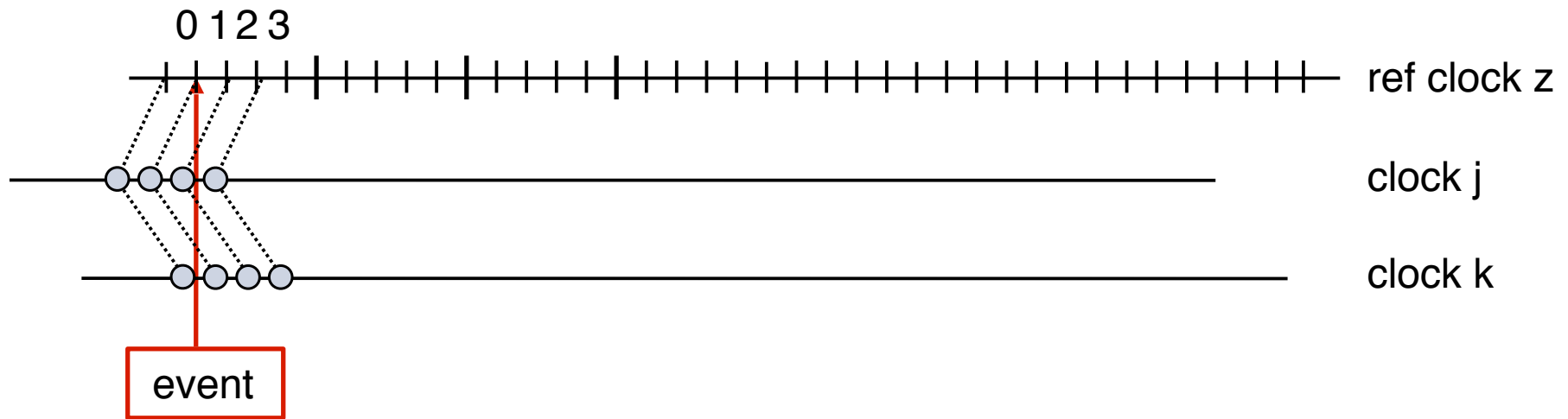
- External Resynchronization
 - resynchronization with reference clock
 - to maintain bounded accuracy
- Internal Resynchronization
 - mutual resynchronization of an ensemble
 - to maintain bounded precision

Global Time

- Given an ensemble of clocks (internally) synchronized with precision π
- For each clock select macrotick as local implementation of a global notion of time with granularity g^{global}
- We note ref clock time (real-time, UTC) in units of g^{global}



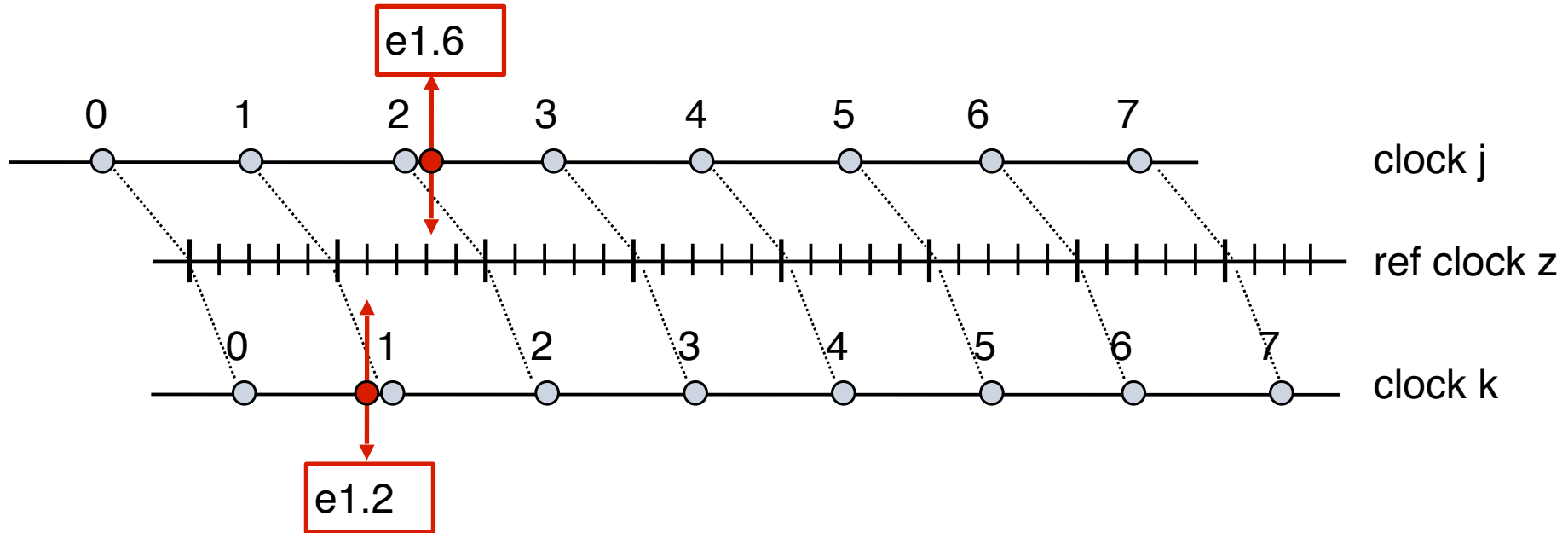
Examples for Bad Choice for Global Time



Reasonable: One Tick Difference

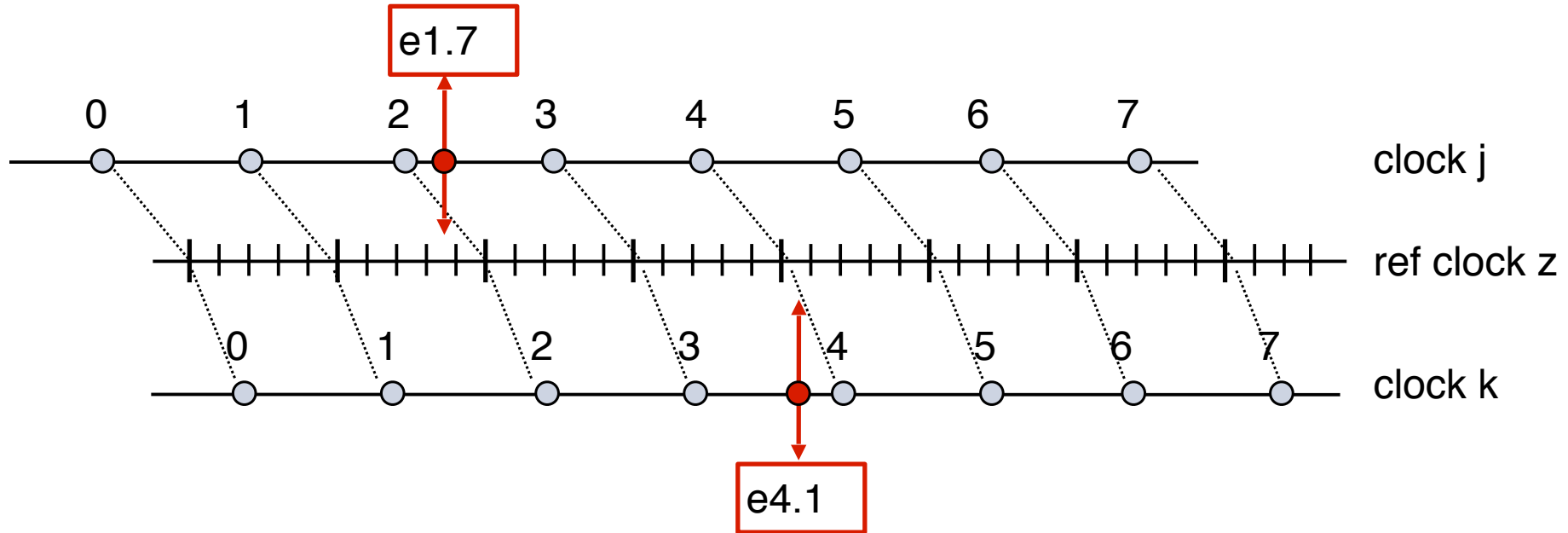
- Reasonableness Condition:
 - global time t is reasonable if $g^{\text{global}} > \pi$ holds for all local implementations
- $t^j(\text{event})$:
 - Denotes global time for the implementation at clock j
- Then: For any single event e , holds: $|t^j(e) - t^k(e)| \leq 1$
- Global timestamps differ at most by one (macro-)tick.
Best one can achieve!

Interpretation for Temporal Order



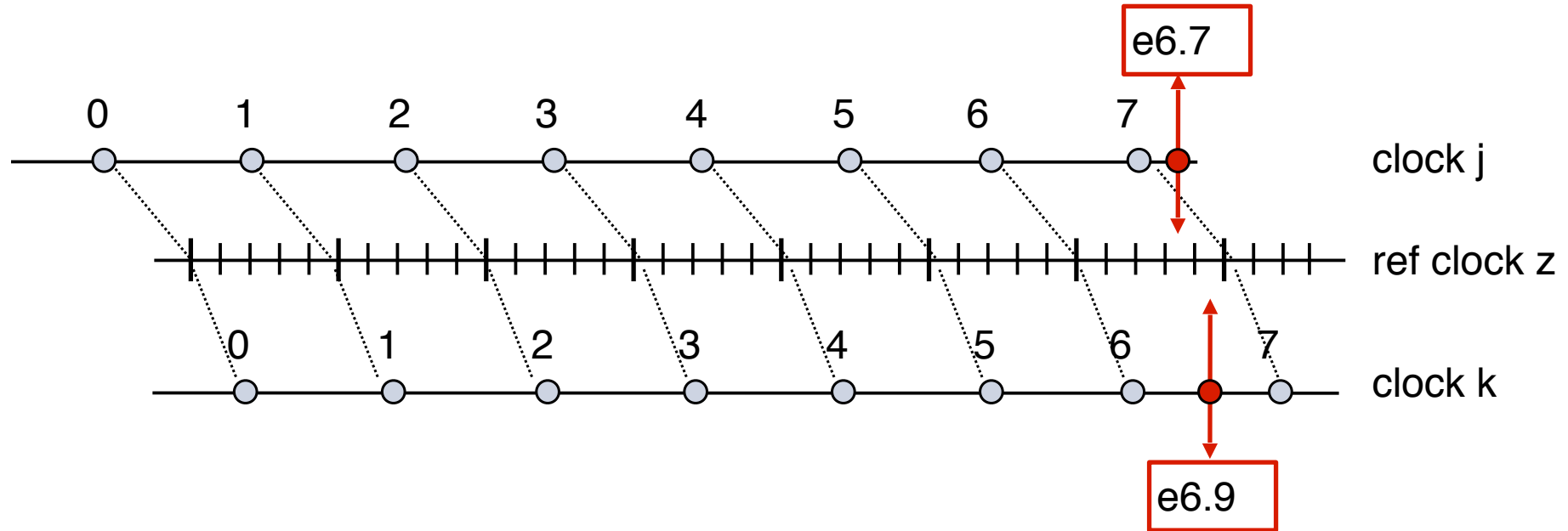
- $z(e1.6) - z(e1.2):$ 0.4 g^{global} ref clock
- $t^j(e1.6) - t^k(e1.2):$ 2 global time ticks
- Temporal order can be established because Tick^k_1 must be before Tick^j_2 (Reasonableness Condition)
- Hence: If the (global) timestamps differ by two ticks, the temporal order can be established.

Caution: Example



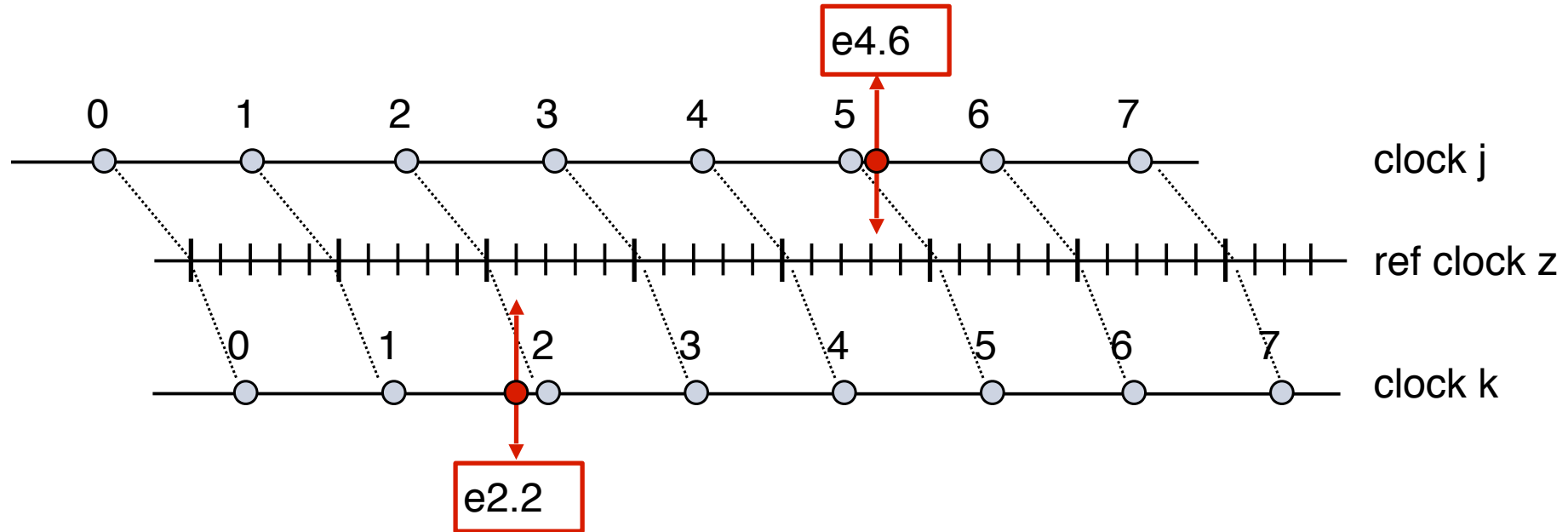
- $z(e4.1) - z(e1.7)$: 2.4 g^{global} ref clock
- $t^k(e4.1) - t^j(e1.7)$: 1 global tick
- A distance of $2 * g^{\text{global}}$ between two events does not suffice to determine temporal order.

Another Example



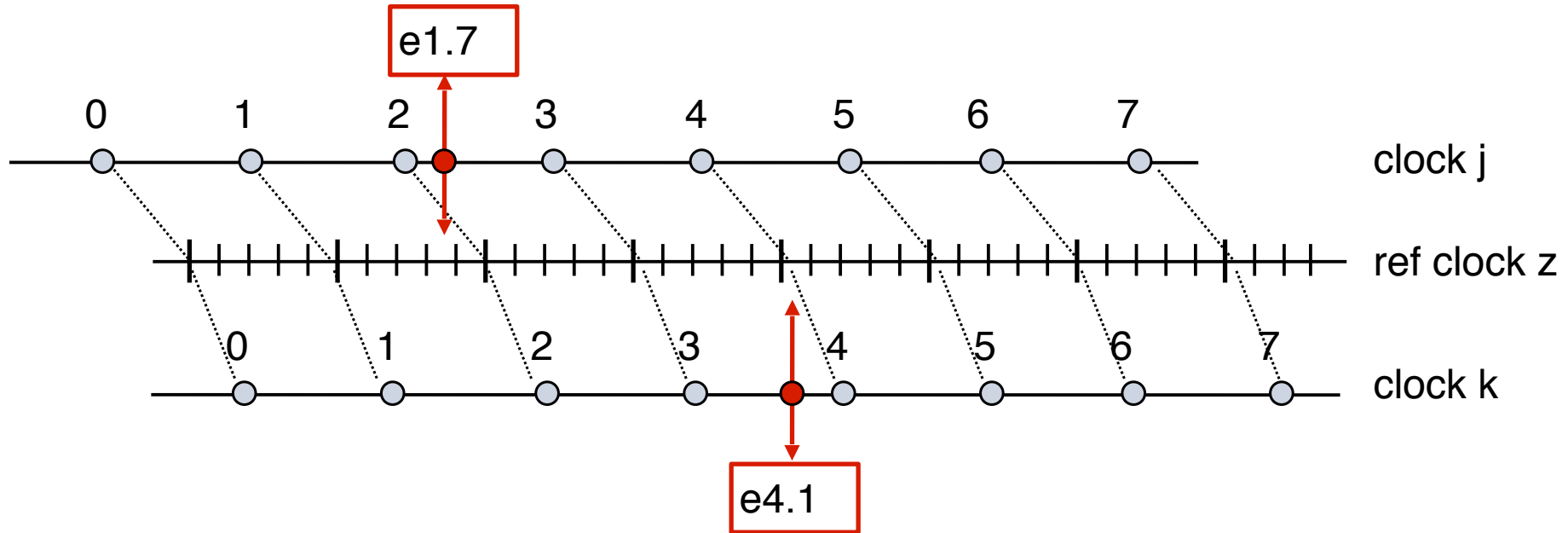
- $z(e6.9) > z(e6.7)$
- $t^k(e6.9) < t^j(e6.7)$

Interpretation for Durations



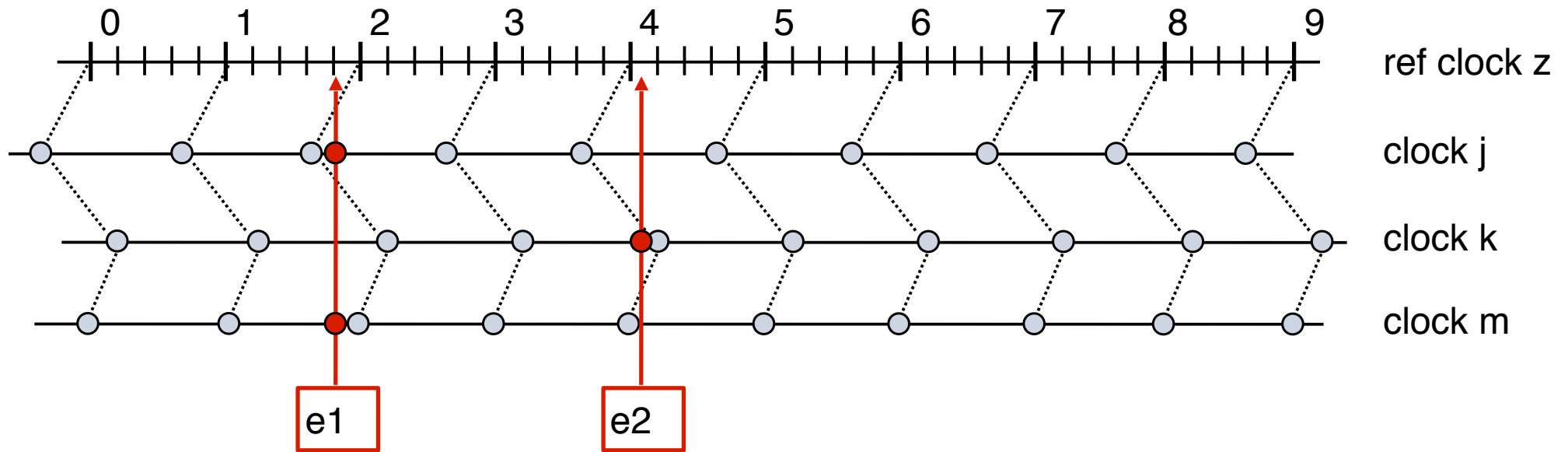
- True duration: 2.4
- Observed duration d : $5 - 1 = 4$ ($t^j(e4.6) - t^k(e2.2)$)
- Can be driven to true duration: $2 + \varepsilon$ for small ε
- $d^{\text{obs}} - 2 * g^{\text{global}} \leq d^z$

Interpretation for Durations



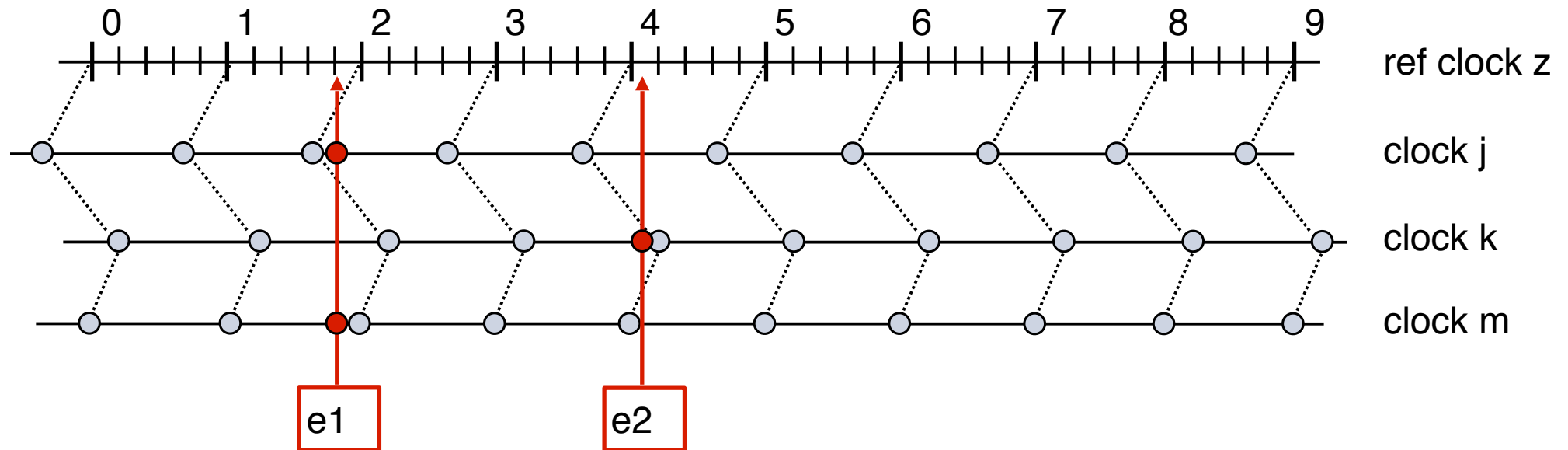
- True duration: 2.4
- Observed duration d : $3 - 2 = 1$ ($t^k(e4.1) - t^j(e1.7)$)
- Example can be constructed for true duration: $3 - \epsilon$ for small ϵ (and still observed duration is 1)
- $d^{\text{obs}} - 2 * g^{\text{global}} < d^z < d^{\text{obs}} + 2 * g^{\text{global}}$

Cooperation and Clocks



- (only) nodes j and m can observe e1
- (only) node k can observe e2
- Node k tells nodes j and m about e2
- Nodes j and m draw their conclusions ...

Dense Time Requires Agreement



- j observes e1 at $t=2$, m observes e1 at $t=1$
- k observes e2 and reports to j and m: “e2 occurred at $t=3$ ”
- j calculates a time difference of 1, hence concludes: “events cannot be ordered”
- m calculates a time difference of 2, hence concludes: “events definitely ordered” $\Rightarrow$ inconsistent view

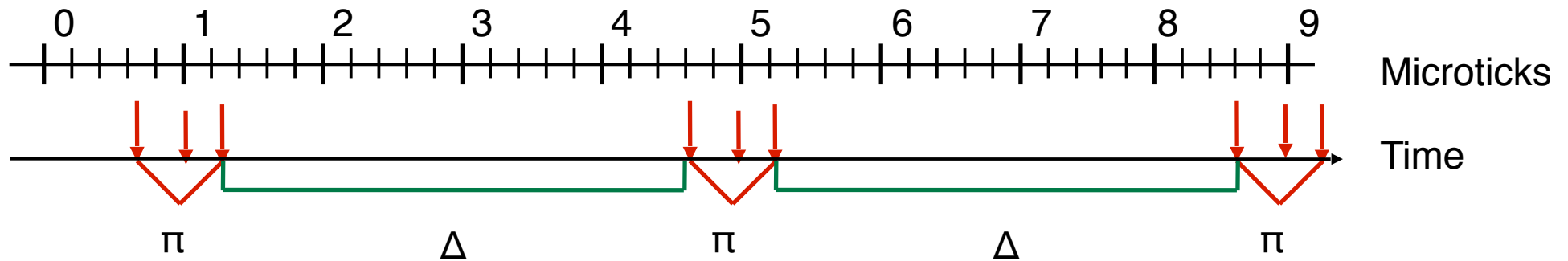
Agreement Protocols

- information interchange:
each node acquires local views from all other nodes
- deterministic algorithm that lead to same result on all nodes
- expensive

Sparse Time

- Two clusters A,B with synch clocks of granularity g each, no clock synch between A and B
- cluster A generates events, cluster B observes
- Goal:
 - if at cluster A events are generated at same cluster wide tick never should temporal order be concluded
 - always establish temporal order otherwise

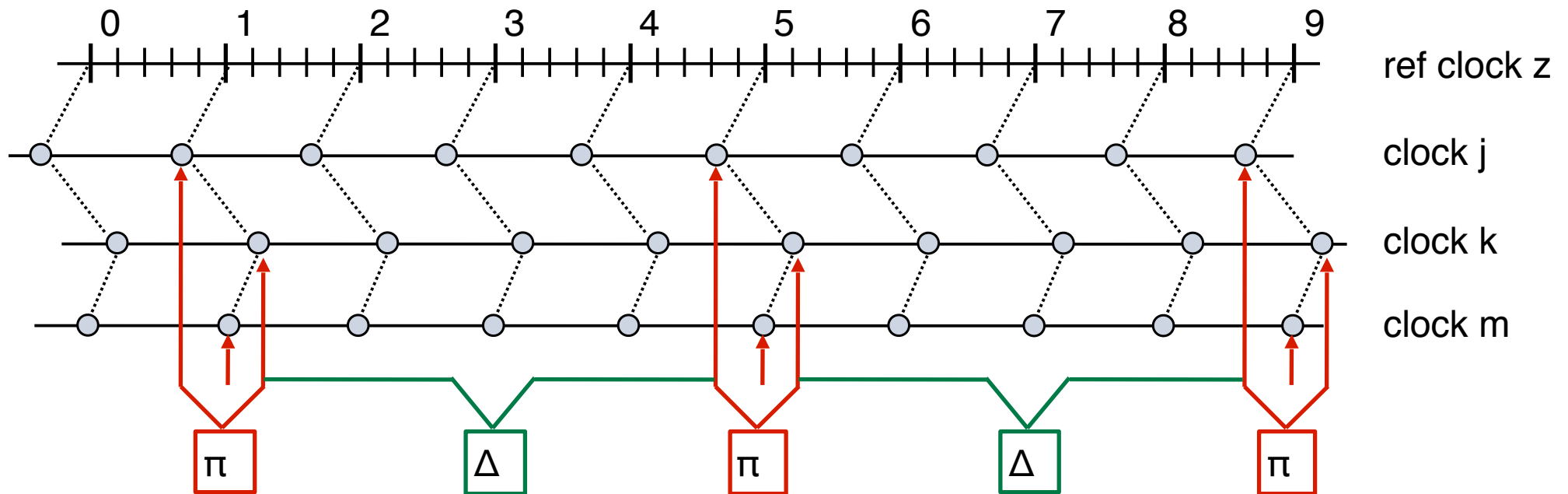
Dense Time vs. Sparse Time



- dense time: events are allowed at any time
- sparse time: events are only allowed within active time intervals π
- sparse time only possible for computer controlled events

Generated Events

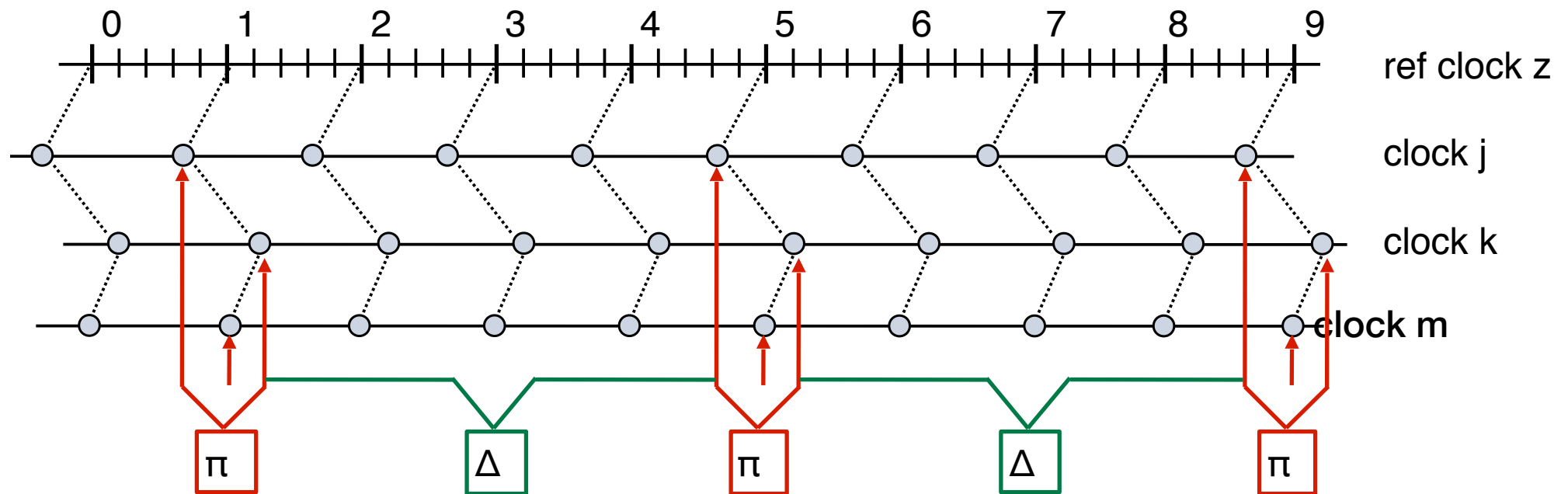
- Cluster of three nodes:
 - each generates event at the same global tick
 - $t = 1, 5, 9$
- observation:



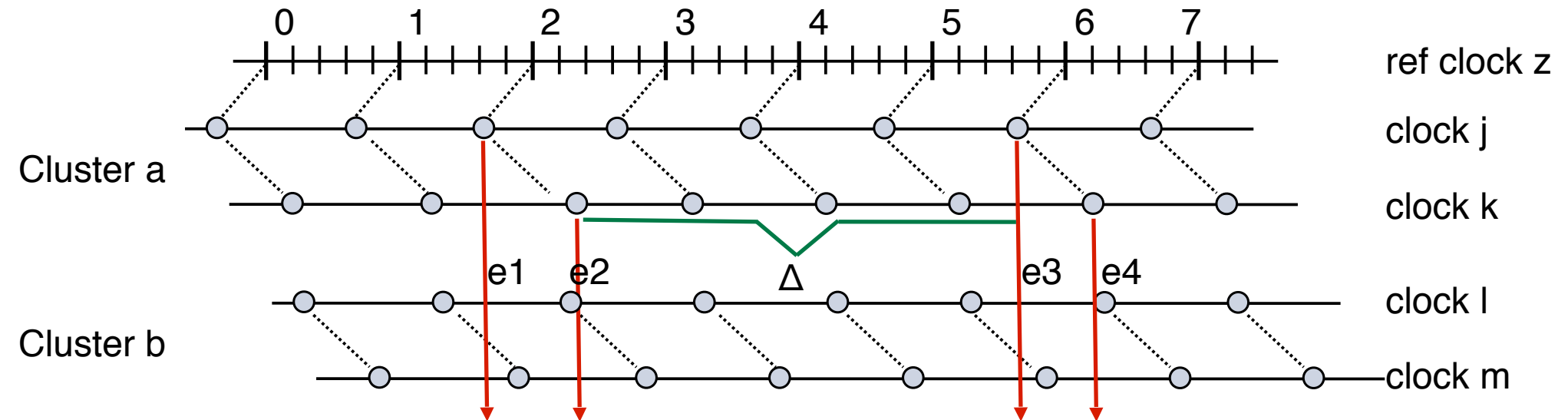
π/Δ -Precedence of Sets of Events

- Properties of sets of events:
 - How far apart (number of granules) must events be to enable reconstruction of order?
- A set of events is called π/Δ -precedent, if:

$$[|z(e_i) - z(e_j)| \leq \pi] \vee [|z(e_i) - z(e_j)| > \Delta]$$



Example for 1g/3g



- $t^l(e2) - t^m(e1) = 2$:
 - BUT: should not derive order because events were intended by cluster A for the same time
- $t^m(e4) - t^l(e2) > 2$ BUT: $t^m(e3) - t^l(e2) = 2$:
 - BUT: temporal order is intended ($\Delta = 3g$)
- $\Rightarrow 1g/3g$ precedence not sufficient $\Rightarrow 1g/4g$

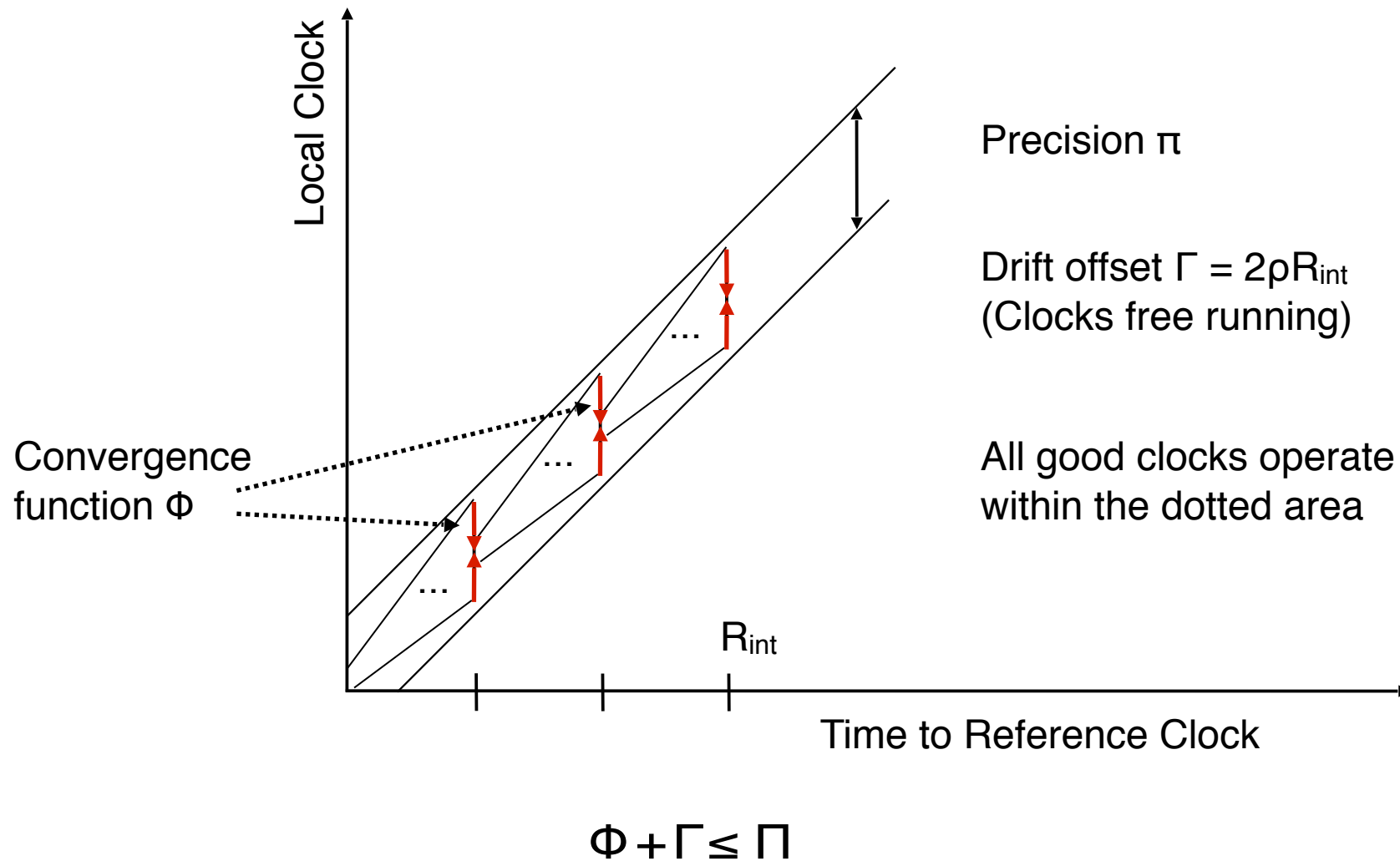
Temporal Order

Event Set	Observed timestamps of two nonsimultaneous events are always greater or equal to	Temporal order of the events can always be reestablished
0/1g precedent	$ t^j(e_1) - t^k(e_2) \geq 0$	no
0/2g precedent	$ t^j(e_1) - t^k(e_2) \geq 1$	no
0/3g precedent	$ t^j(e_1) - t^k(e_2) \geq 2$	yes
0/4g precedent	$ t^j(e_1) - t^k(e_2) \geq 3$	yes

Fundamental Results in Time Measurement

- A single event observed at different nodes may have timestamps differing by one; not sufficient to establish temporal order
- Duration: $d^{\text{obs}} - 2 \cdot g^{\text{global}} < d^z < d^{\text{obs}} + 2 \cdot g^{\text{global}}$
- temporal order can be recovered from their timestamps if they differ by 2 global time ticks
- temporal order of events can always be recovered from timestamps if the event set is 0/3g precedent

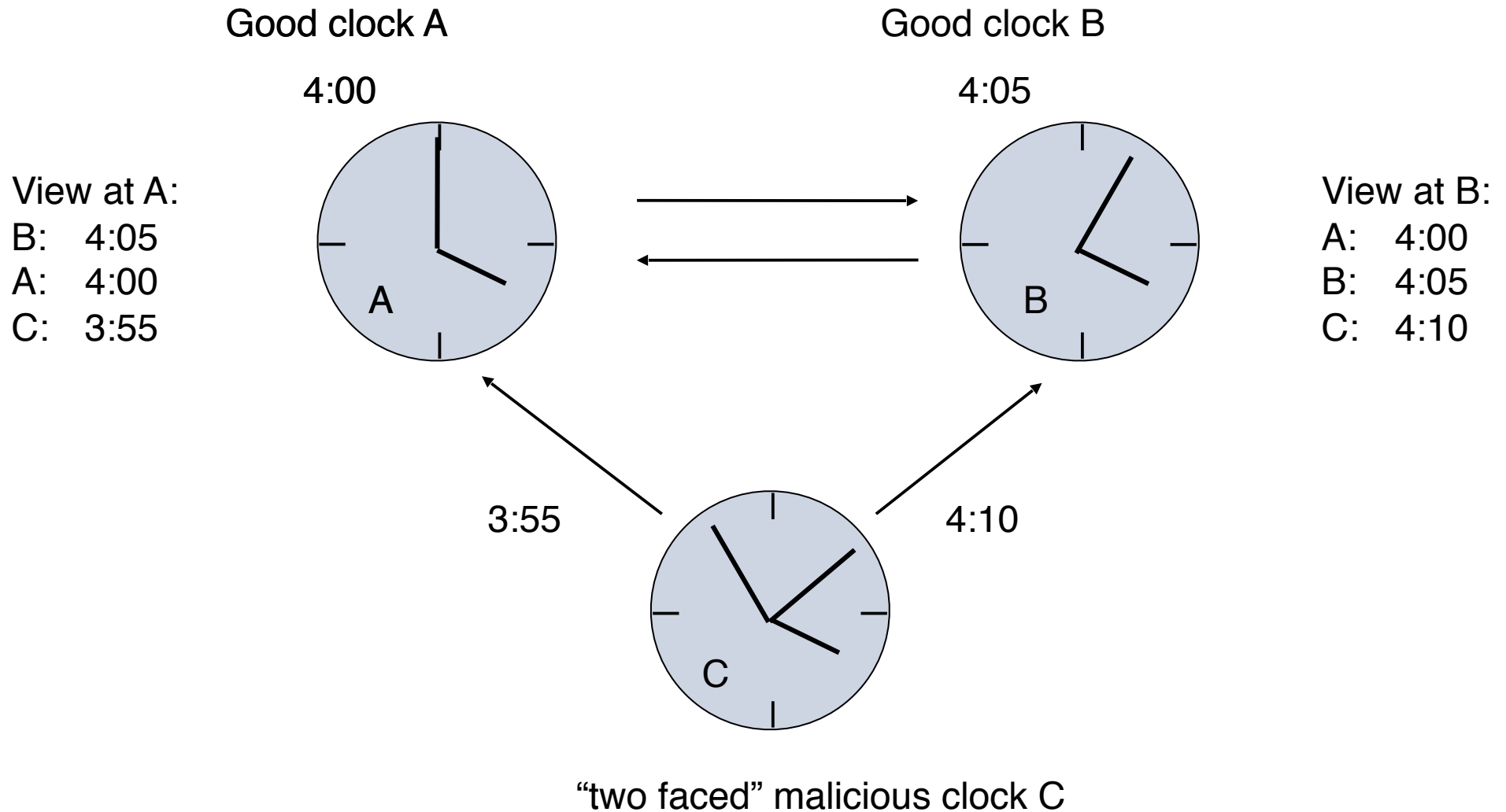
Internal Clock Synchronisation



Synchronisation Condition

- resynchronization interval: R_{int}
- convergence function : Φ offset after resynch.
- drift offset: Γ
- Required: $\Gamma = 2 \rho R_{\text{int}}$
 $\Gamma + \Phi \leq \Pi$

Byzantine Error



Impossibility Result

$$\pi = \varepsilon \left(1 - \frac{1}{N} \right)$$

- No better precision can be achieved even with perfect clocks in all nodes (N number of nodes).

- Logical Clocks:
Standard text books: Coulouris, Tanenbaum
- Physical Clocks:
This lecture followed strictly
Hermann Kopetz, Distributed Real-Time-Systems
- David Mills: Internet Time Synchronisation: the Network Time Protocol, IEEE Transactions Communic. 39,10 (Oktober 1991)