Real-Time Systems

Hard Real-Time Multiprocessor Scheduling

Marcus Völp



Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling (no migration)
- Global Scheduling (task- / job-level migration)
 - G-FP
 - G-EDF
- Optimal MP Scheduling
- MP Resource Access Protocols
- Open Research Issues

Lessons Learned From UP

- Liu-Layland Criterion for fixed task priority algorithms:
 - can schedule any workload up to a utilization of $U_{RMS}(n) = n\sqrt[n]{2} 1 \le 0.693$
 - scheduling of workloads with higher utilization not guaranteed
- fixed job priority algorithms are optimal: (e.g., EDF)
- there are optimal greedy algorithms
 - with a single measure characterizing the "importance" of a job (e.g., time to deadline, laxity, ...)
- all preemptive FTP, FJP algorithms are predictable
 - response times cannot increase when decreasing execution times
- all preemptive FTP algorithms and EDF are sustainable
 - no period / deadline anomalies
- simultaneous release is critical instant
- response times depend on <u>set</u> but not on order of high-prio. tasks

Taxonomy of Multiprocessor Scheduling

Two problems to solve:

Priority Problem: When to run a given job of the workload?

fixed task priority (e.g., RMS, ...)

fixed job priority (e.g., EDF, ...)

- dynamic job priority (e.g., LST, ...)

Allocation Problem: Where to run this job?

no migration

- task-level migration (no migration of running jobs)

- job-level migration (migrate also running jobs)

partitioned

global

Taxonomy of Multiprocessor Scheduling

Two problems to solve:

- Priority Problem: When to run a given job of the workload?
 - fixed task p
 - fixed job pri Preemption Costs UP / MP
 - dynamic job
- Allocation Problem: Where to run this job?
 - no migration
 - task-level m
 - job-level mig

Migration Costs – MP

partitioned

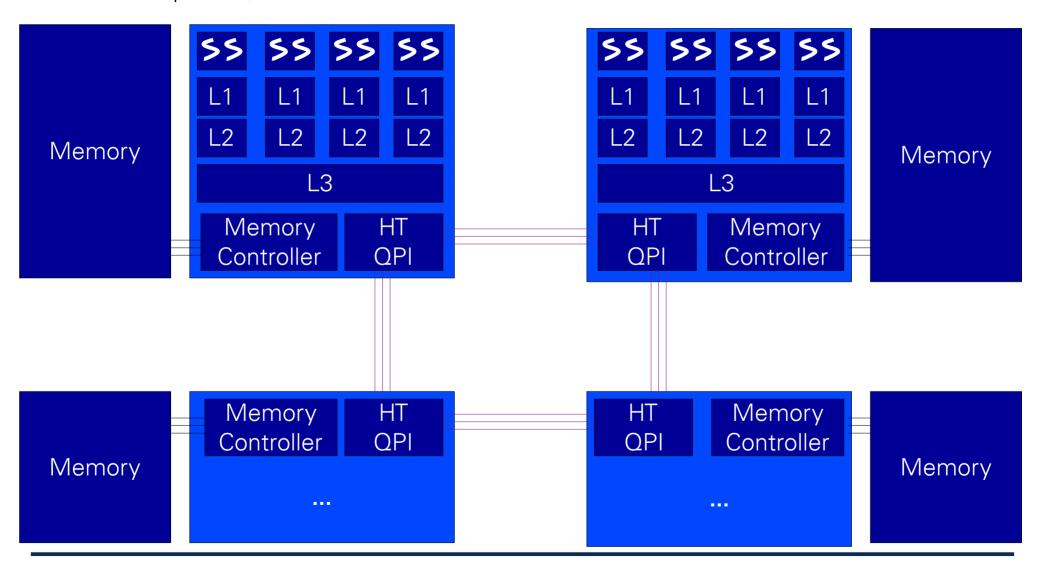
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Preemption Costs

- direct costs:
 - timer / device interrupt
 - save register state
 - manipulate ready list
 - UP: no synchronization required
 - load register state of next thread
- indirect costs
 - cache evictions between two consecutive runs
 - TLB refills after evictions / shootdown

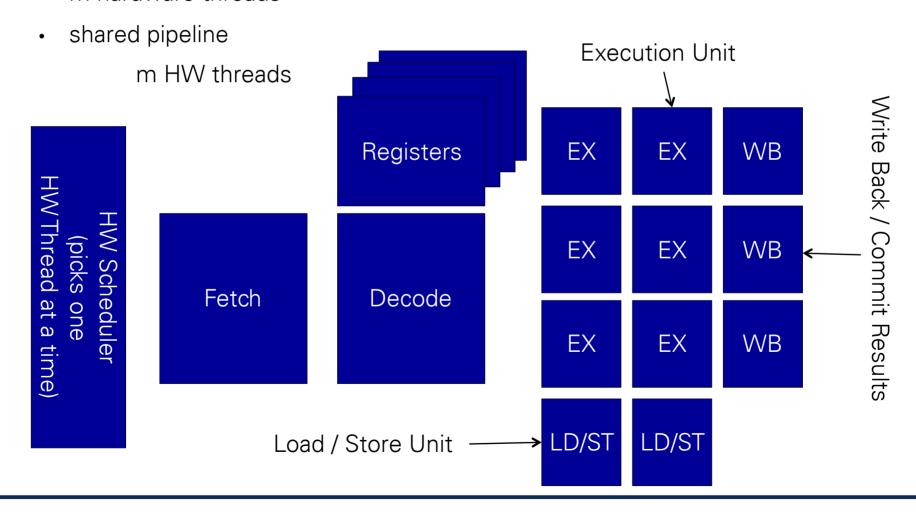
- job-level migration
 - migration of running job implies preemption at source CPU
- task-level migration
 - job is already preempted
- direct costs
 - manipulate remote / global ready list
 - synchronization
 - fetch register state
- indirect costs
 - fetch active cache working set from remote cache
 - load remaining data from remote memory

AMD Opteron / Intel Core Duo: SMT + multi core + ccNuma

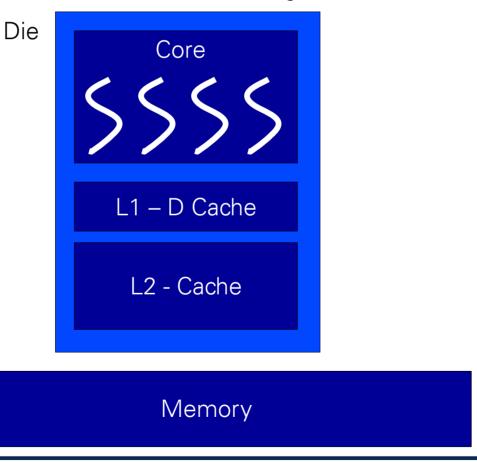


- Symmetric Multi-Threaded (SMT) Processors
 - m hardware threads
 - shared pipeline m HW threads Registers EX EX WB HW Thread at a time) HW Scheduler (picks one EX EX WB Fetch Decode EX EX WB LD/ST LD/ST

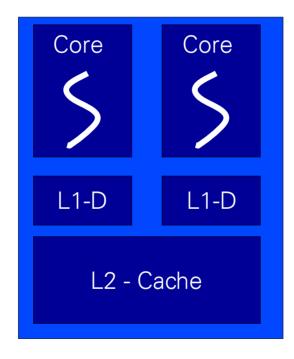
- Symmetric Multi-Threaded (SMT) Processors
 - m hardware threads



- Symmetric Multi-Threaded (SMT) Processors
 - operating system multiplexes n SW threads on m HW threads
 - caches + pipeline is shared => no indirect migration costs

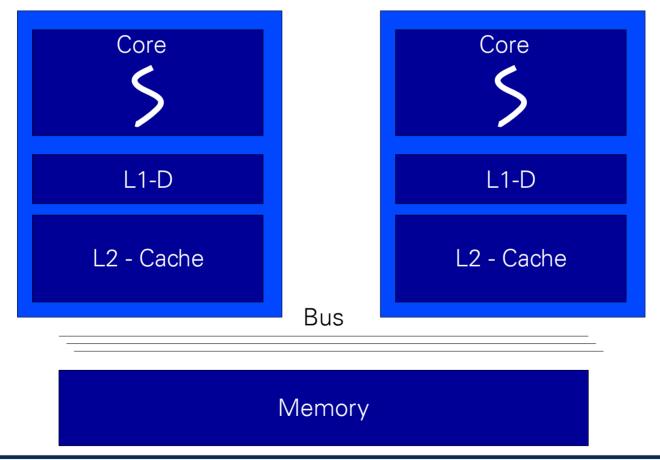


- Multi-Core Processors
 - operating system multiplexes n SW threads on m cores
 - timing of last level cache dominates migration costs

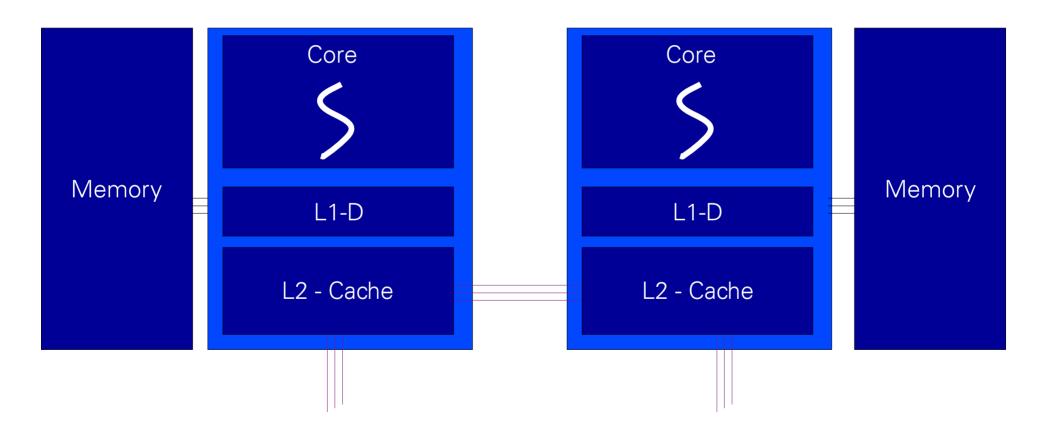


Memory

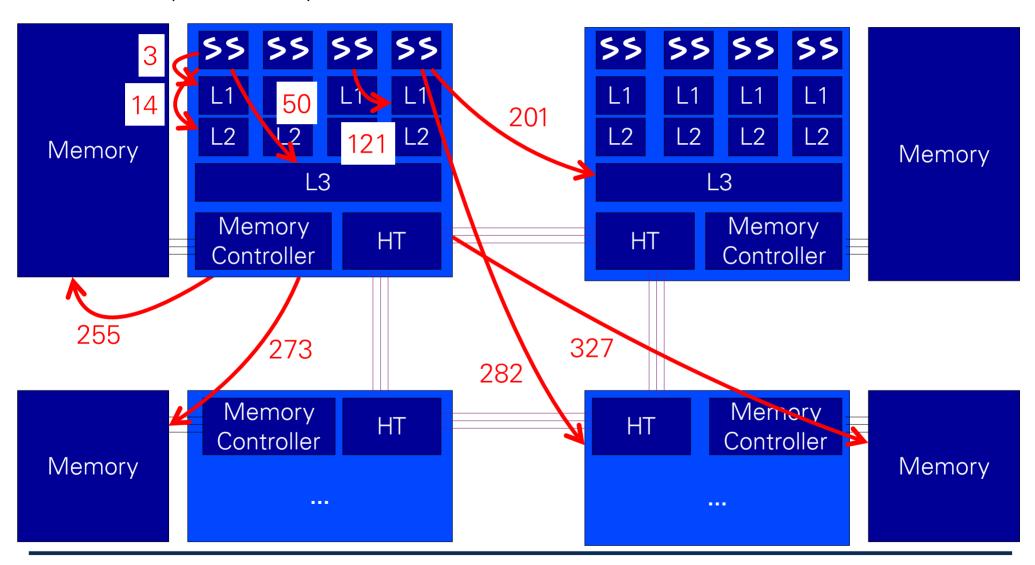
- Symmetric Multiprocessors
 - operating system multiplexes n SW threads on m dies
 - timing of interconnect dominates migration costs



- (cache coherent) NUMA
 - like SMP
 - non-uniform memory access: fetch from remote memory



AMD Opteron [Corey: OSDI '08]



- Active Cache Working Set
 - · cachelines a thread would access again if it would run
 - · varies over time
 - ages out after preemption

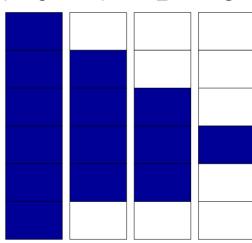
ways:

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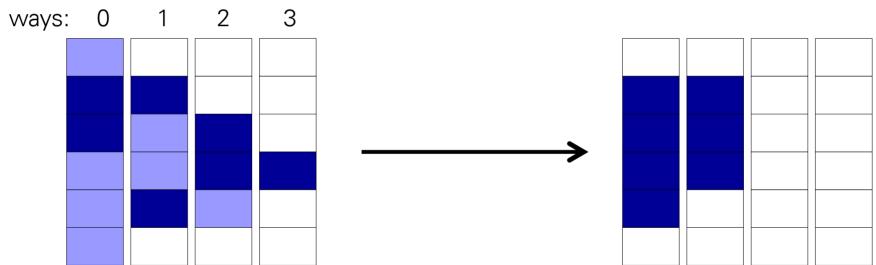
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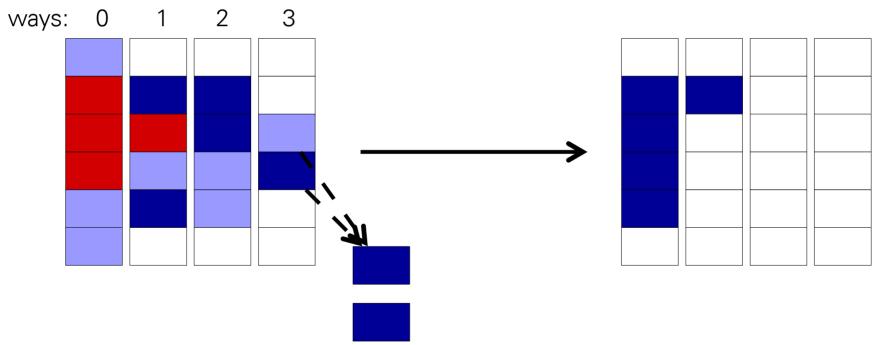
3



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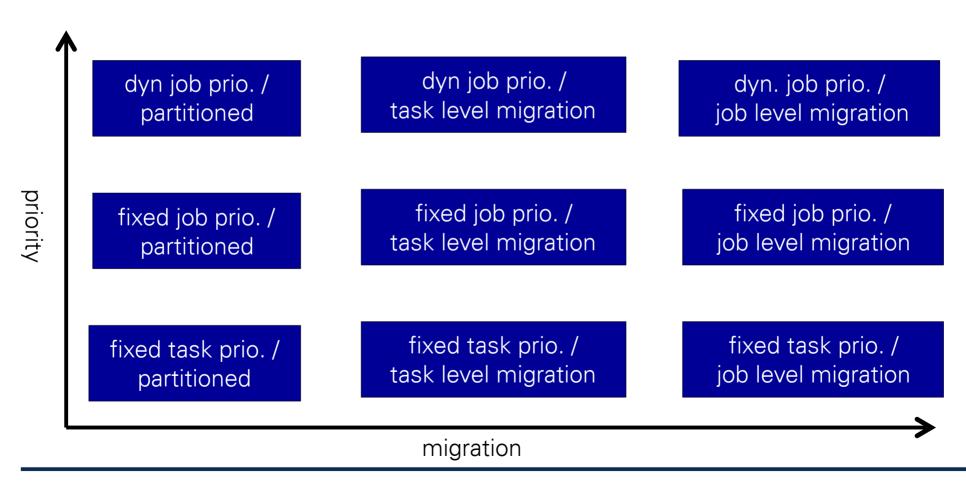


- Active Cache Working Set
 - · cachelines a thread would access again if it would run
 - varies over time
 - ages out after preemption



- Summary
 - migration costs are highly architecture dependent
 - non-trivial to predict
 - may cause a significant delay when a thread resumes execution
- Assumption for the remainder of this lecture:
 - zero preemption and migration costs / attributed to WCET

Design Space of MP Scheduling



Design Space of MP Scheduling

Partitioned

dyn job prio. / partitioned

fixed job prio. / partitioned

fixed task prio. / partitioned

Global

dyn job prio. / task level migration

fixed job prio. / task level migration

fixed task prio. / task level migration

dyn. job prio. / job level migration

fixed job prio. / job level migration

fixed task prio. / job level migration

Design Space of MP Scheduling

Relative ordering between classes of scheduling algorithms

dyn. job prio. / job level migration

dyn job prio. / task level migration

fixed job prio. / partitioned

dyn job prio. / partitioned

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Later in this lecture

Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling
- Global (Task-Lvl migration) Scheduling
 - G-FTP (e.g., G-RMS)
 - G-EDF
- Optimal MP Scheduling
- MP Resource Access Protocols
- Open Research Issues

Terminology, Notation and Assumptions

- Periodic Tasks
 - Task $t_i = (P_i, D_i, C_i)$
- $P_i = \text{const.}$

- Sporadic Tasks
 - P_i = Minimal Interrelease Time
- Deadlines
 - implicit deadline: $D_i = P_i$

(relative deadline = period)

(deadline may be after period end)

- constrained: D_i :
 - $D_i \le P_i$ (relative deadline < period)

arbitrary

 $U_i = \frac{C_i}{P_i}$

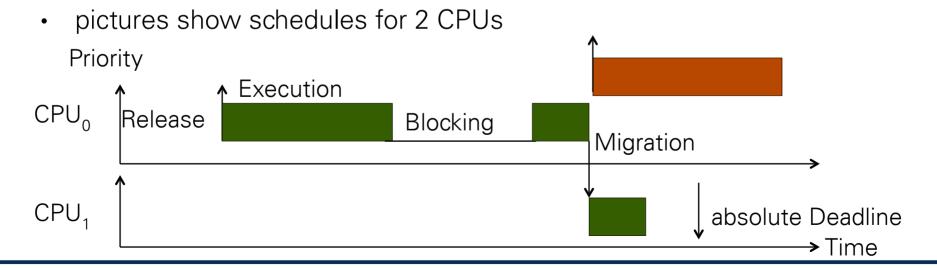
Utilization

 $\partial_i = \frac{C_i}{\min(D_i, P_i)}$

Density

Terminology, Notation and Assumptions

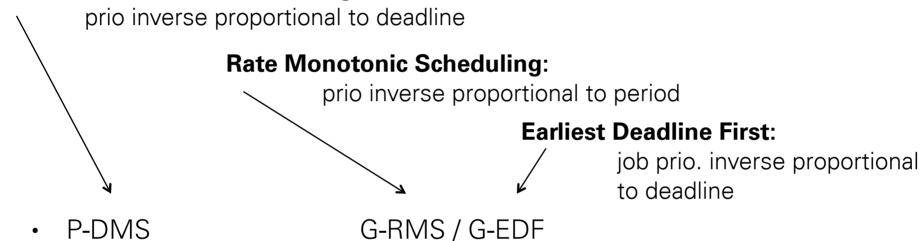
- Assumptions for the remainder of this lecture
 - independent tasks
 - fully preemptible / migratable (negligible costs)
 - unlimited number of priorities
 - tasks are single threaded: a job can utilize only 1 CPU at a time
 - jobs do not block (shared resources later in this lecture)



Terminology: P-DMS / G-RMS / G-EDF

Scheduling Algorithms:

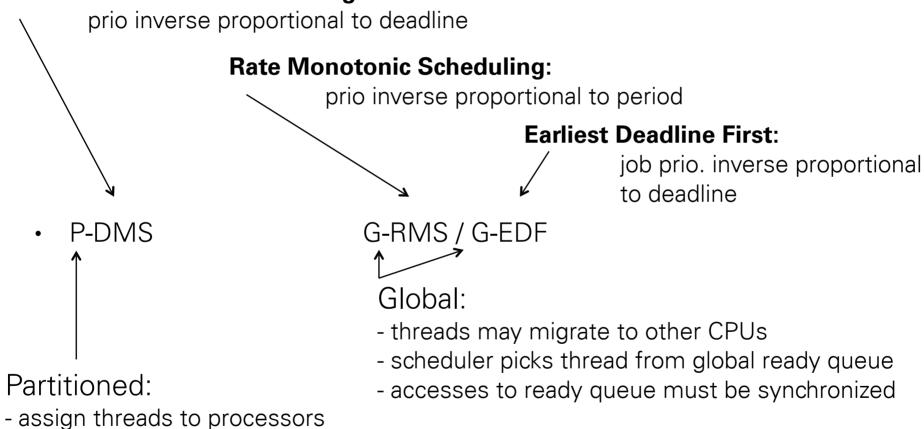




Terminology: P-DMS / G-RMS / G-EDF

Scheduling Algorithms:

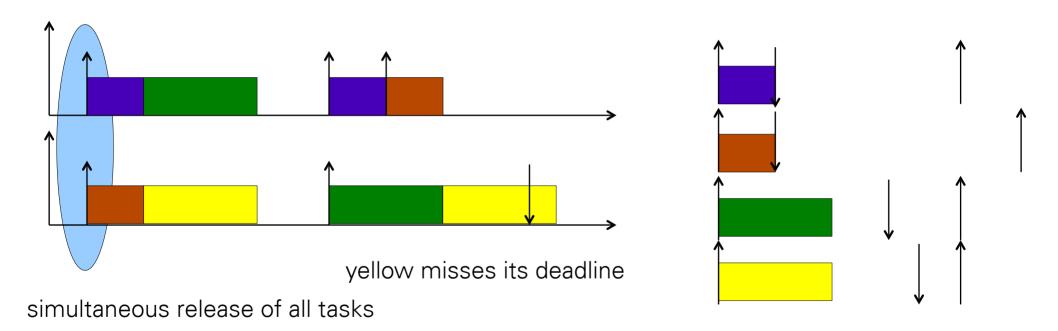
Deadline Monotonic Scheduling:



- scheduler picks threads from local (per CPU) ready queue
- no synchronization overhead for accessing the ready queue

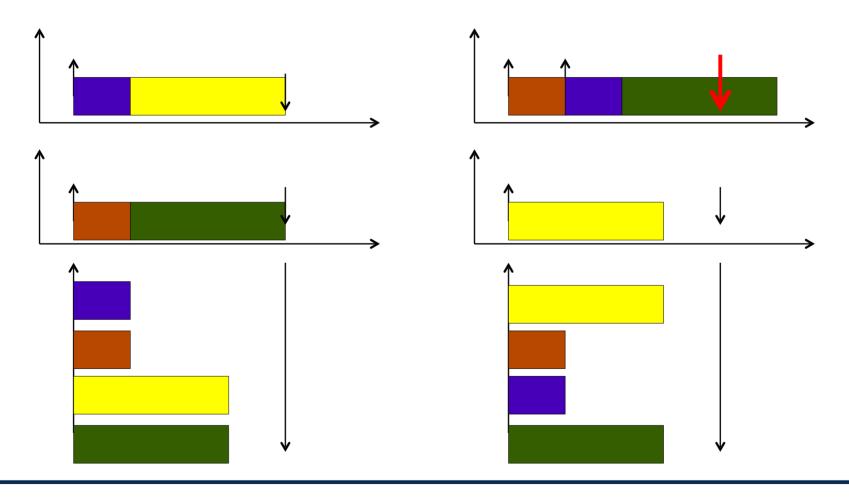
Anomalies

- Simultaneous Release is not Critical Instance [Lauzac '98]
 - longer response time in second period



Anomalies

 Response time (of green) depends not only on set of higher prioritized tasks but also on their relative <u>priority ordering</u>



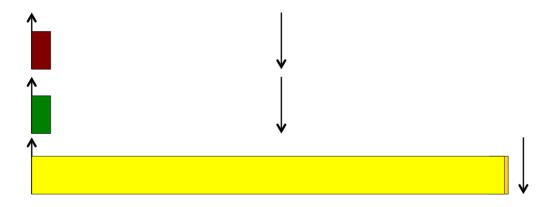
Sustainability [Baruah '06]

- A schedulable workload remains schedulable if we
 - decrease the execution time of a task (predicability)
 - otherwise, WCET won't work as admission criterion
 - increase the minimal interrelease time (period) of a task
 - otherwise, more frequent recurrence is no safe approximation
 - increase the relative deadline of a task
 - otherwise, earlier deadline is no safe approximation
- G-FTP + G-EDF are not sustainable if #CPUs > 1
- all preemptive FJP / FTP algorithms are predictable



Dhall Effect

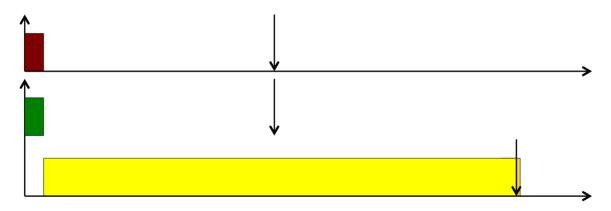
- The utilization bound of Global EDF is as low as $U_{EDF} = 1 + \epsilon$
 - m tasks with short periods and infinitesimal low U_i (e.g., $U_i = \varepsilon$)
 - 1 task with larger period and U_j close to 1 (e.g., $U_j > (2 \epsilon) / 2$)



Dhall Effect does not manifest if U_i < 41 %

Dhall Effect

- The utilization bound of Global EDF is as low as $U_{EDF} = 1 + \epsilon$
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Dhall Effect does not manifest if U_i < 41 %

some Impossibility Results

- [Hong '88]
 - No optimal online MP scheduling algorithm for arbitrary collections of jobs, unless all jobs have the same relative deadline.
- [Dertouzos '89]
 - Even if execution times are known precisely, clairvoyance for job arrivals is necessary for optimality.
- [Fisher '07]
 - No optimal online algorithm for sporadic tasksets with constrained or arbitrary deadlines.

Partitioned Scheduling

Partitioned

dyn job prio. / partitioned

fixed job prio. / partitioned

fixed task prio. / partitioned

dyn job prio. / task level migration

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dyn. job prio. / job level migration

fixed job prio. / job level migration

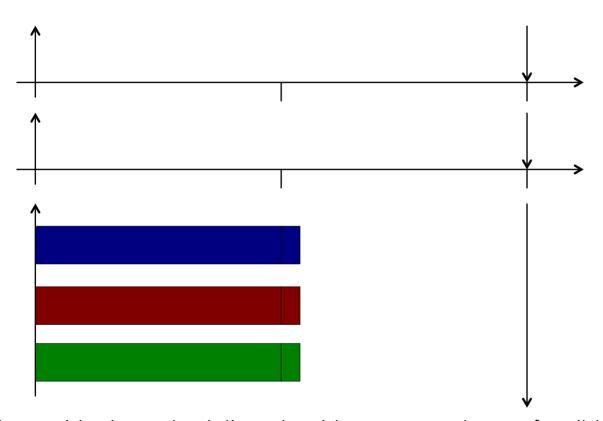
fixed task prio. / job level migration

Partitioned Scheduling

- Split workload by allocating tasks to CPUs
- Run allocated task with UP scheduling algorithm
 - reap benefit of well known UP results
 - optimal task allocation is NP complete:
 - pack n tasks with density d_i on m CPUs with capacity $d_{max} = 1$
 - Bin-packing

Partitioned Scheduling

Utilization bound for implicit deadline workloads [Anderson '01]

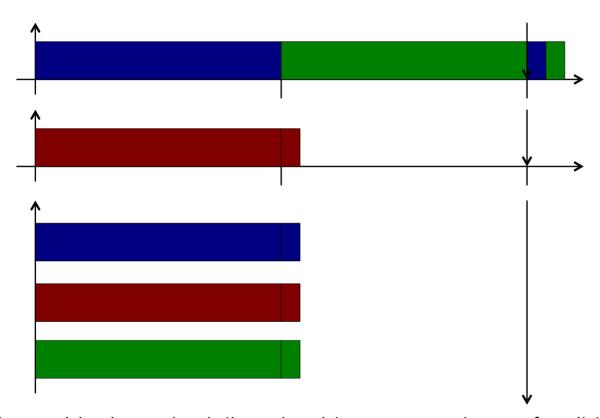


$$U_{opt} = \frac{m+1}{2}$$

No partitioning scheduling algorithm can produce a feasible schedule of m+1 tasks with execution time 1+e and period of 2 on m processors

Partitioned Scheduling

Utilization bound for implicit deadline workloads [Anderson '01]

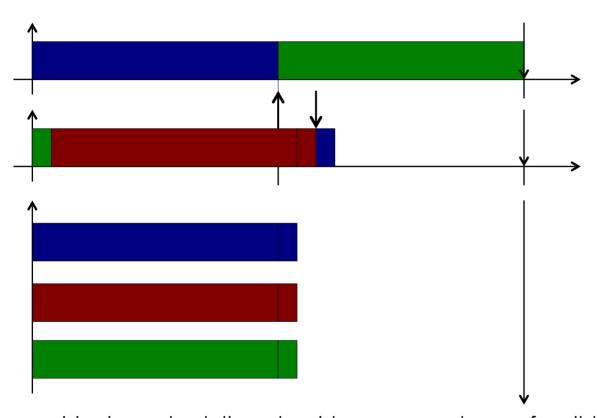


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$$U_{opt} = \frac{m+1}{2}$$

No partitioning scheduling algorithm can produce a feasible schedule of m+1 tasks with execution time 1+e and period of 2 on m processors

Easy if blue and green can migrate to CPU₂

PDMS-HPTS-DS [Lakshmanan '09]

- Can we improve on Anderson's Utilization Bound?
 - by allowing a few jobs to migrate
- PDMS Partitioned Deadline Monotonic Scheduling
- HPTS Split Highest Priority Task
- DS Allocate according to Highest Density First

CPU₀

$$\tau_{1} = (4,3,1)$$

$$u_{1} = 0.25 \ \delta_{1} = 1/3 = 0.33$$

$$\tau_{2} = (6,2,2)$$

$$u_{2} = 0.33 \ \delta_{2} = 1$$

$$u_{3} = 0.25 \ \delta_{3} = 1/4 = 0.25$$

$$\tau_{4} = (6,4,2)$$

$$u_{4} = 0.33 \ \delta_{4} = 1/2 = 0.5$$

$$u_{5} = 0.16 \ \delta_{5} = 1/5 = 0.2$$

$$u_{8um} = 1.33 \implies u_{8um} / 2 = 0.66\%$$

$$\begin{array}{c} \tau_1 = (4,3,1) \\ \tau_2 = (6,2,2) \\ \tau_3 = (4,4,1) \\ \tau_5 = (6,5,1) \\ \end{array} \qquad \begin{array}{c} u_1 = 0.25 \ \delta_1 = 1/3 = 0.33 \\ u_2 = 0.33 \ \delta_2 = 1 \\ u_3 = 0.25 \ \delta_3 = 1/4 = 0.25 \\ u_4 = 0.33 \ \delta_4 = 1/2 = 0.5 \\ \hline u_5 = 0.16 \ \delta_5 = 1/5 = 0.2 \\ \hline u_{\text{sum}} = 1.33 \ \Rightarrow u_{\text{sum}} \ / \ 2 = 0.66\% \end{array}$$

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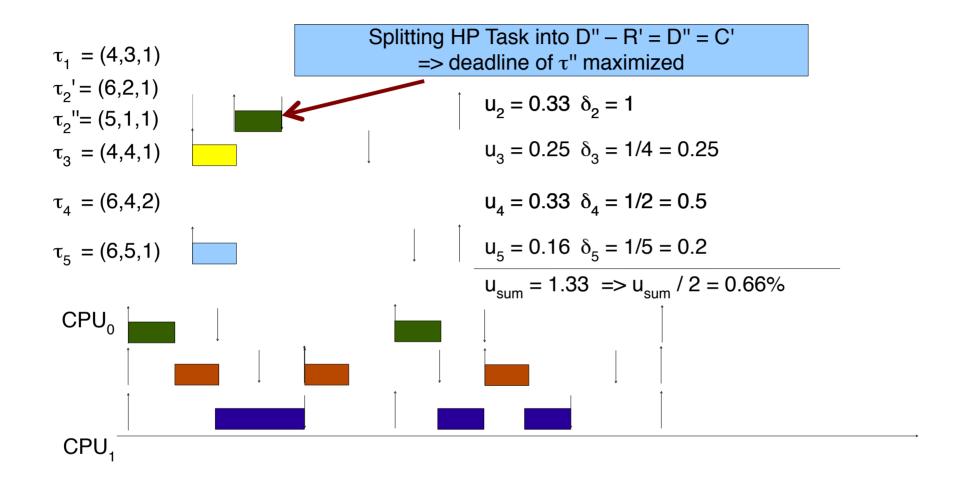
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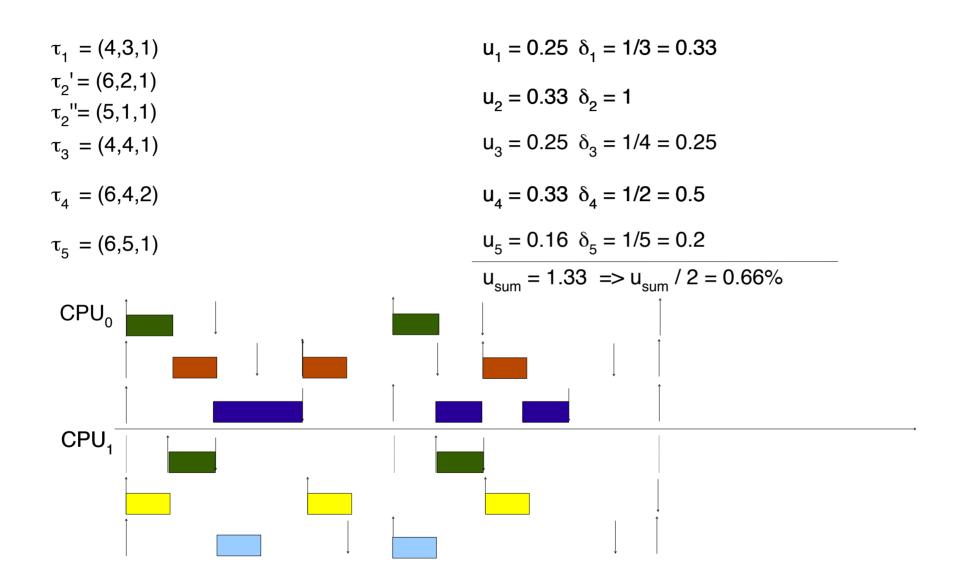
$$u_{4} = 0.33 \ \delta_{4} = 1/2 = 0.5$$

$$u_{5} = 0.16 \ \delta_{5} = 1/5 = 0.2$$

$$u_{8um} = 1.33 \implies u_{8um} / 2 = 0.66\%$$

$$CPU_{0}$$





• $U_{PDMS-HPTS-DS} = 69.3 \%$ if all tasks have a utilization $U_i < 41\%$

Global Scheduling

Global

dyn job prio. / partitioned

fixed job prio. / partitioned

fixed task prio. / partitioned

dyn job prio. / task level migration

fixed job prio. / task level migration

fixed task prio. / task level migration

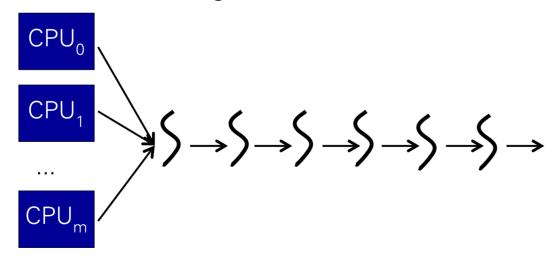
dyn. job prio. / job level migration

fixed job prio. / job level migration

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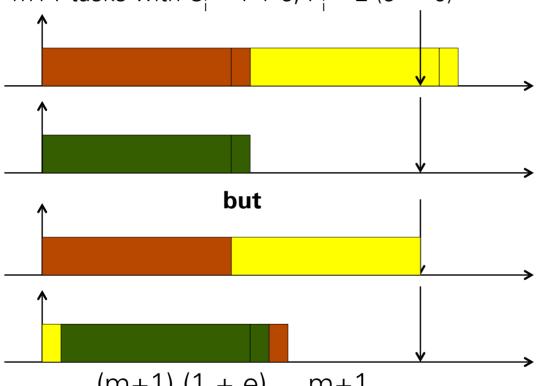
Global Scheduling

- Always pick the ready jobs of the m most "important" tasks
- A task may migrate
 - When a new job of a task is released it may receive a different CPU; once started, a job is no longer migrated
- No need for allocation / load balancing
 - Load balancing is automatic



Global Scheduling – Utilization Bound

- Utilization bound for global fixed-job priority algorithms
 - on m CPUs, G-FJP algorithms cannot schedule m+1 tasks with $C_i = 1 + e$, $P_i = 2$ ($e \rightarrow 0$)



•
$$U_{OPT} = \lim_{e \to 0} \frac{(m+1)(1+e)}{2} = \frac{m+1}{2}$$

Global Scheduling - Job Level Migration

dyn job prio. / partitioned

dyn job prio. / task level migration

dyn. job prio. / job level migration

fixed job prio. / partitioned

fixed job prio. / task level migration

fixed job prio. / job level migration

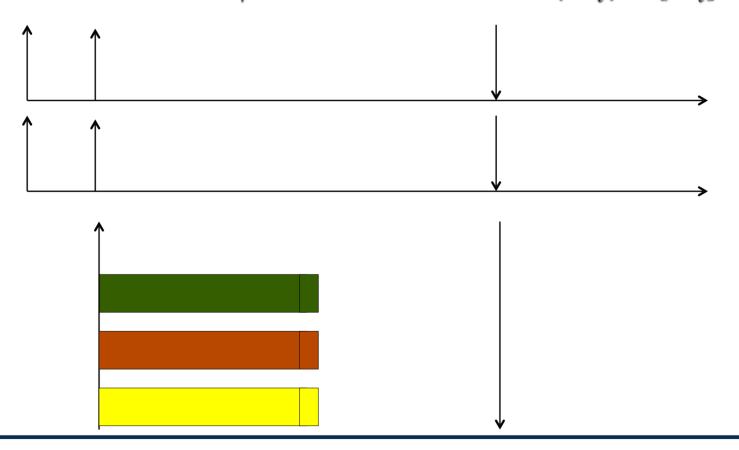
fixed task prio. / partitioned

fixed task prio. / task level migration

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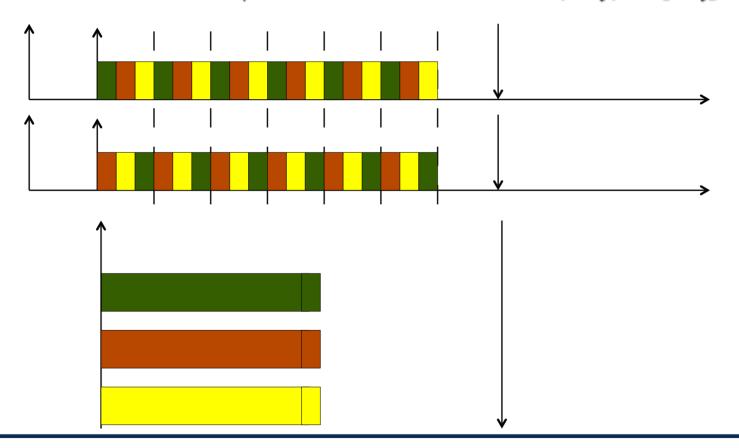
PFAIR [Baruah '96]

- Divide timeline into equal length quanta
- At each quanta of length t, allocate tasks to processors such that the accumulated processor time is either [tui] or [tui]



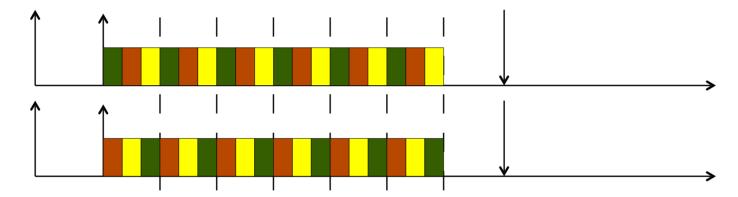
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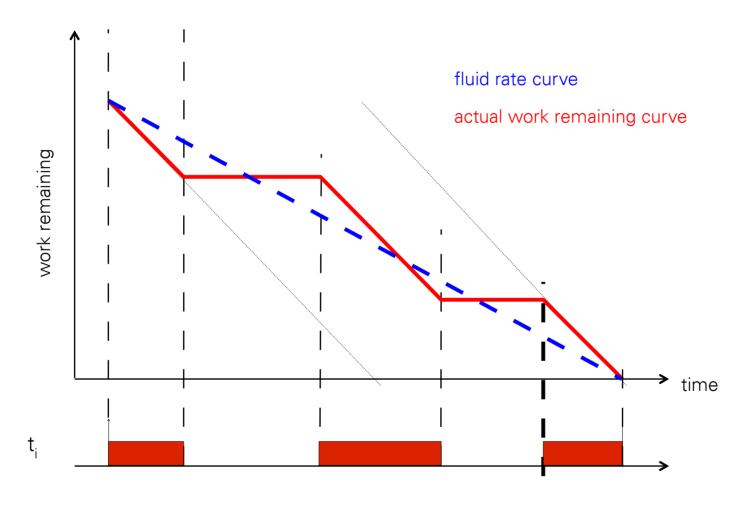


- PFAIR is optimal for periodic implicit deadline tasksets: $U_{OPT} = m$
- Very high preemption and migration costs

- DP-Fair
 - Optimal scheduler for periodic implicit deadline tasksets with a minimal number of preemptions / migrations?
 - Recall [Hong '88]:
 - No optimal MP scheduling algorithm for arbitrary tasksets if not all tasks have the same relative deadline

- deadline partitioning
 - any task's deadline becomes a deadline for all tasks
- always run zero laxity jobs
 - laxity = time to deadline remaining execution time
 - zero laxity => job may miss its deadline if it is not run
- jobs that twine themselves around the fluid rate curve are somehow in good shape

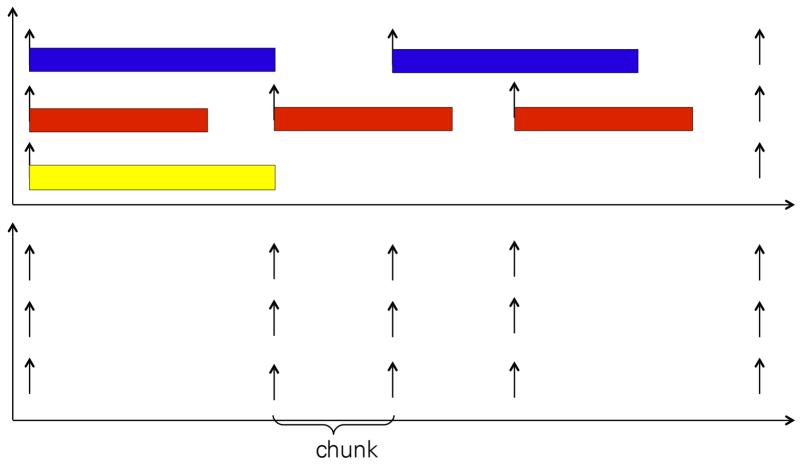
Fluid Rate Curve



zero laxity event: no more time to run others

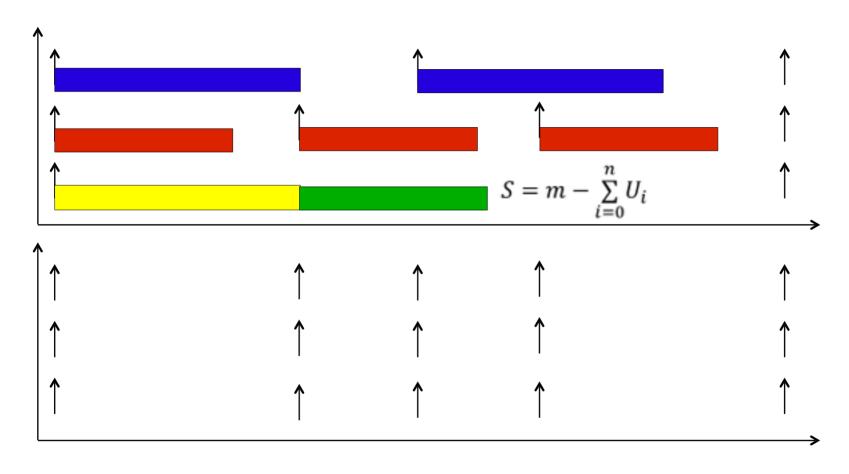
- DP-Fair:
 - a family of optimal, deadline partitioning scheduling algorithms
 - Split timeline into chunks according to job deadlines
 - Allocate work to a chunk proportional to U_i
 - local execution time: $C_{i,j}^{l}=(t_{j+1}-t_{j})\ U_{i}$
 - Rule 1: always run a job with zero local laxity
 - jobs with remaining local execution time = time to end of chunk
 - Rule 2: never run a job with no remaining local work
 - Rule 3: split up idle time proportional to length of chunk
 - allocate at most (m U_{sum}) ($t_{j+1} t_j$) idle time to chunk j

Deadline partitioning



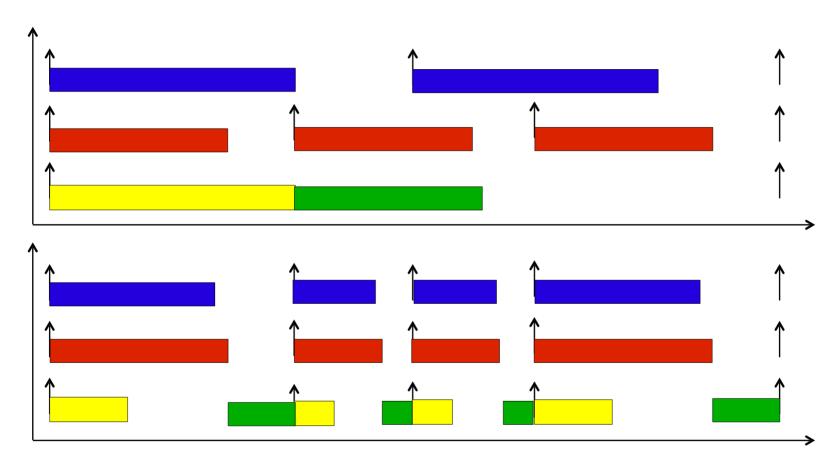
Introduce additional releases / deadlines for all jobs whenever there is such an event for one job in the original schedule => chunks

Idle time



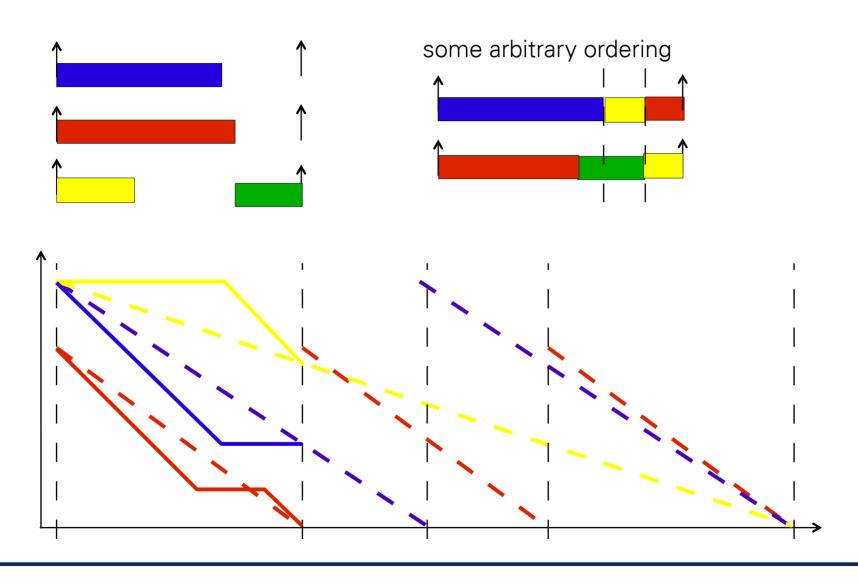
Treat idle time as just another job to schedule.

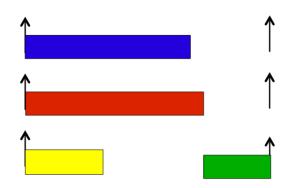
Allocate work proportional to Ui



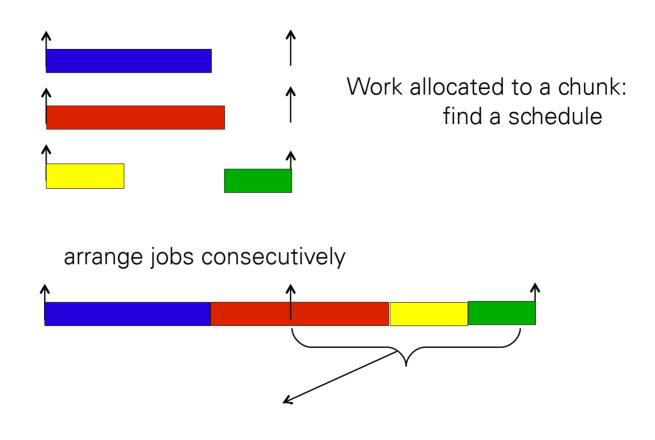
Allocate execution (and idle) time of a job proportionally to its utilization => amount of time that this job must run in a given chunk

Jobs hit their fluid rate curve at the end of each chunk

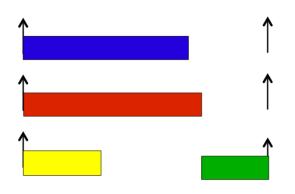




Work allocated to a chunk: find a schedule

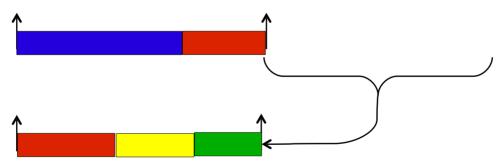


wrap around to obtain schedule for 2nd CPU, ...



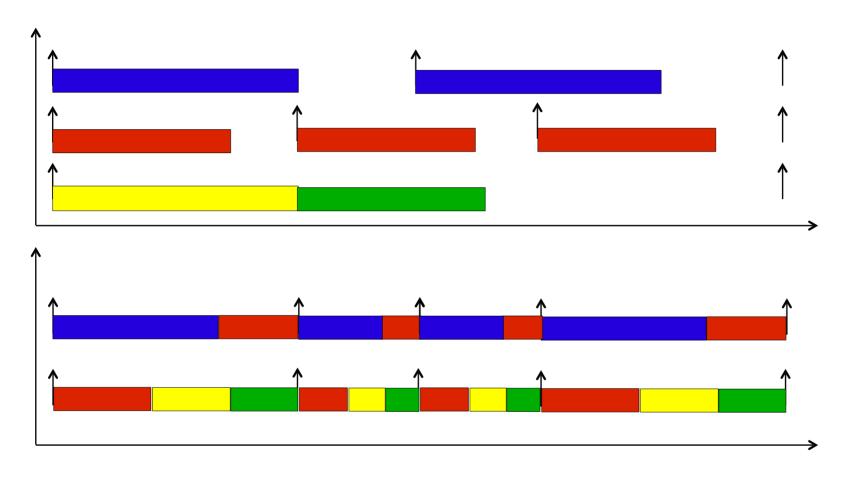
Work allocated to a chunk: find a schedule

arrange jobs consecutively



wrap around to obtain schedule for 2nd CPU, ...

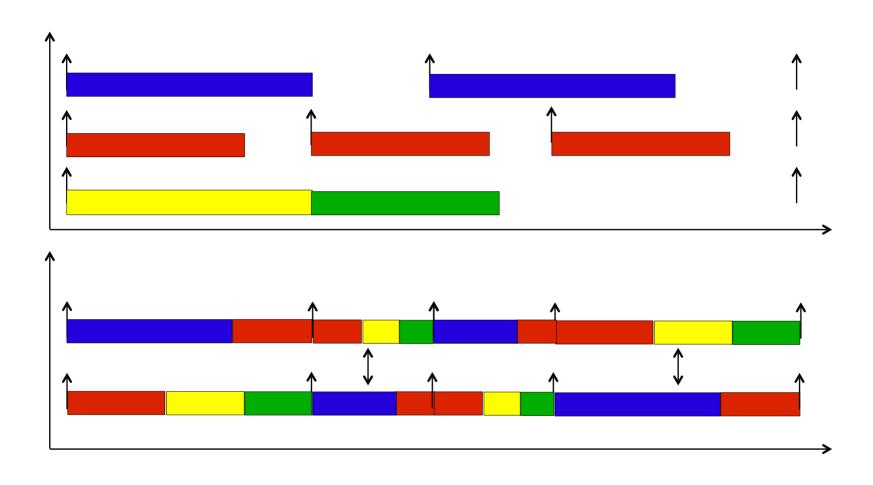
- m 1 migrations per chunk
- n 1 context switches per chunk



Unnecessary migration of red task at chunk boundaries

=> mirror processor assignment of every second chunk

DP-Wrap (mirrored)



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dyn job prio. / partitioned

dyn job prio. / task level migration

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job level migration

fixed job prio. / partitioned

fixed job prio. / task level migration

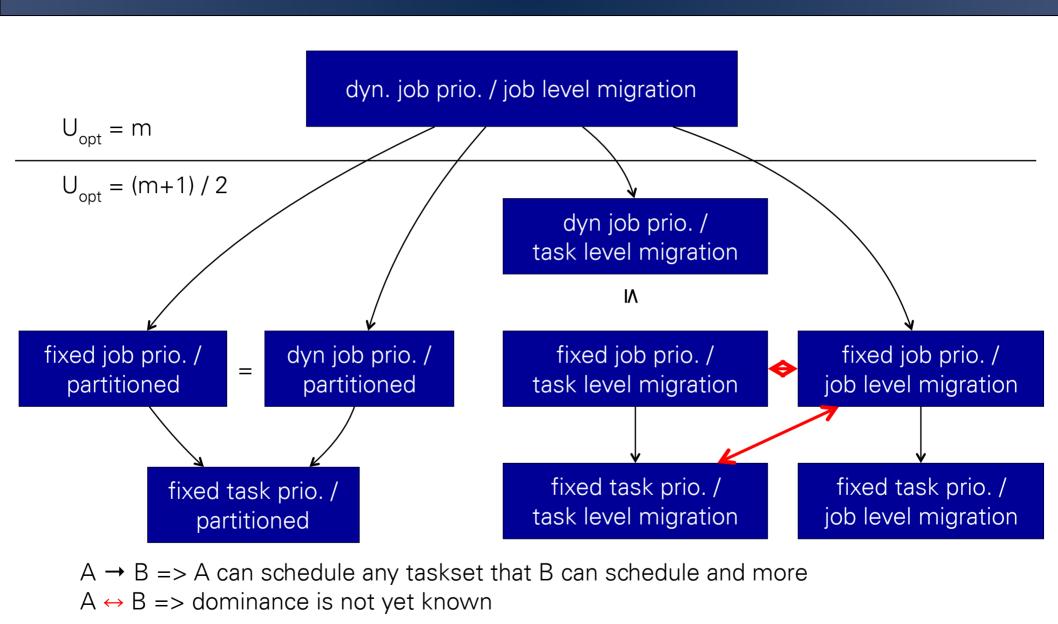
fixed job prio. / job level migration

fixed task prio. / partitioned

fixed task prio. / task level migration

fixed task prio. / job level migration

Design Space of MP Scheduling



Outline

- Introduction
- Terminology, Notation and Assumptions
- Anomalies + Impossibility Results
- Partitioned Scheduling
- Global (Task-Lvl migration) Scheduling
 - G-FTP (e.g., G-RMS)
 - G-EDF
- Optimal MP Scheduling
- MP Resource Access Protocols
- Open Research Issues

MP Resource Access Protocols

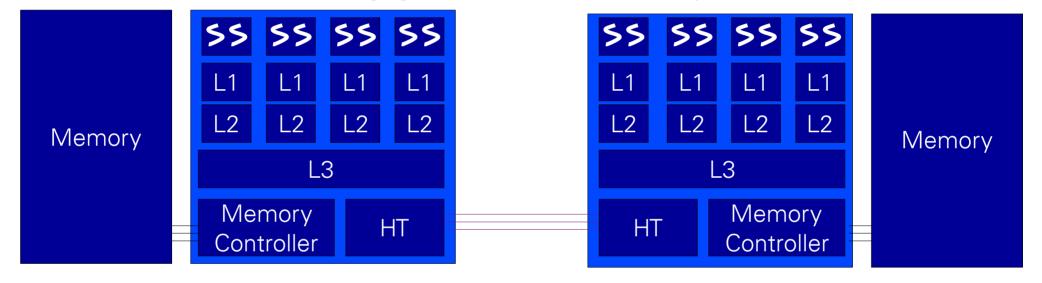
- UP-BPCP in a nutshell
 - resource holders inherits priority from blocked threads
 - resource granted if $prio(t_i) > \hat{S}$
 - only the resource holder, which holds a resource with $\hat{\mathbf{R}}_i = \hat{\mathbf{S}}$ receives additional resources
- UP-SRP (actually UP-CPP) in a nutshell
 - resource holder runs at max $\hat{\mathbf{R}}_{i}$ of held resources => only higher prioritized threads may run (acquire resources)

MP Resource Access Protocols

- UP:
 - Basic Priority Ceiling Protocol BPCP
 - Stack Resource Protocol SRP (Ceiling Priority Protocol CPP)
 - bounded priority inversion: | CS |
 - BPCP does not influence unrelated threads
- General Idea:
 - run UP protocol on every CPU of MP system
- Ceiling Priority i of Resource R_i : $\hat{\mathbf{R}}_i = \max \text{prio}(t_j)$
 - here, priorities have a global meaning
- System Ceiling $\hat{S} = \max \hat{R}_i$ of held resources
- Synchronization Processor: CPU on which R_i is executed

Locking for Clustered Scheduling

- [Brandenburg '11]:
 - clustered scheduling: global within the cluster; partitioned in between



- Idea:
 - Every task helps out resource holders for a bounded time
 - Only the n-highest prioritized threads may acquire resources

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Open Issues [Burns '09]

- Limited processor Utilization
 - minimally dynamic algorithms
 - novel partitioning approaches
 - · increase the guaranteed processing capability; overheads
- Ineffective Schedulability Tests (in particular, sporadic workloads)
 - large gap between feasibility / infeasibility tests
 - identify finite collection of worst-case job arrival sequences
- Overheads
 - migration costs; run queue manipulations; context switching
 - algorithms that permit intra-cluster migration; task-level migr.

Task Models

- intra-task parallelism, runtime integration
- heterogeneous resources, Turbo Boost, GPUs

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